Fourth International Workshop on Reliable Engineering Computing

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Robust Design - Coping with Hazards, Risk and Uncertainty

A Novel Robust Control Strategy for Interval Plants Using The Two Loop MFC and CDM

by

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Layout

- Introduction
- Two Loop Control Structure
- Coefficient Diagram Method
- Proposed Control Structure
- Design Examples
- Conclusions

Robust Control

- Satisfactory Controller performance for a family of plants.

- The controller ensures stability and performance despite

    1. Disturbances and
    2. Modeling uncertainties.

Control Strategies of interest

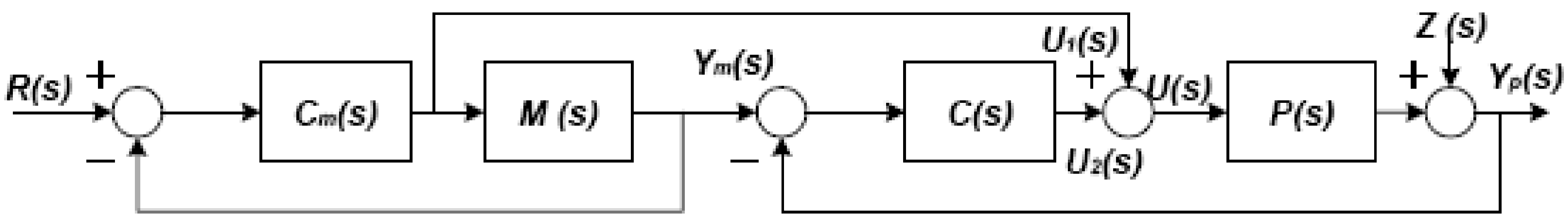
1. Nominal Model Following Control

2. Coefficient Diagram Method

Nominal Model Following Control (NMFC)

- Excellent model following property (Tracking)
- Two degree freedom of control gives design flexibility

NMFC Structure



$R(s)$ – The reference input

$Cm(s)$ – The model controller

$Ym(s)$ – The reference output

$P(s)$ – The real plant

$P(s) = M(s)(1 + \Delta(s))$

$\Delta(s)$ – Multiplicative perturbation

$M(s)$ – The nominal plant

$U_1(s)$ – The model controller output

$U_2(s)$ – The robust controller output

$Y_p(s)$ – The real plant output

$Z(s)$ – The output disturbance

Key Features of NMFC

- The real plant output tracks the nominal plant output
    1. In the absence of the modeling errors
    2. In the presence of the modeling errors

- The controllers for both the loops can be designed independently using any technique

Conventional Vs NMFC Control Structures

Control Structure	Nature of Control	Tracking
Conventional	single degree freedom of control	Poor model following property
NMFC	Two degree freedom of control	Excellent model following property

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Control Structure	Relative Stability	Effect of parameter variations
Conventional	poor	more sensitive
NMFC	good	less sensitive

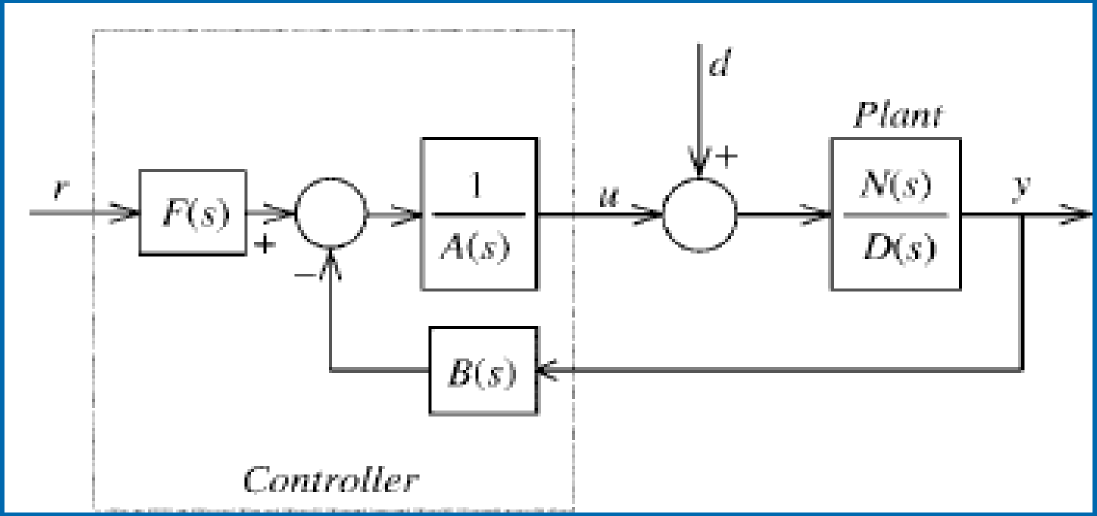
Coefficient Diagram Method

- One of the recently developed controller design methodologies
- An algebraic approach combining classical control and modern control design
- The design avoids pole zero cancellations

Key Features of CDM

- Simultaneous design of Characteristic polynomial and controller
- Main Design tool: Coefficient Diagram
- Stability, response speed and robustness analyzed in one diagram

Basic Structure



r : the reference input,

y : the output,

u : the control and

d : the external disturbance

F(s) : the reference numerator polynomial

N(s) and D(s) : plant polynomials

A(s) : the forward denominator polynomial of the controller

B(s) : the feedback numerator polynomial of the controller

Design Parameters

For a characteristic polynomial of the form

$$P(s) = a_n s^n + \dots + a_1 s + a_0 = \sum_{i=0}^{n} a_i s^i$$

- The equivalent time constant τ

Indicates Response speed

$$\tau = \frac{a_1}{a_0}$$

- The stability indices γ_i are defined in terms of coefficients of the characteristic polynomial

$$\gamma_i = \frac{a_i^2}{a_{i+1} a_{i-1}}$$

Indicate stability

- The stability limits γ_i^* are defined in terms of stability indices

$$\gamma_i^* = \frac{1}{\gamma_{i-1}} + \frac{1}{\gamma_{i+1}}$$

Stability Conditions

- The sufficient condition for stability is given as

$$\gamma_i > 1.12\gamma_i^*, \text{ for all } i = 2, \ldots n-2$$

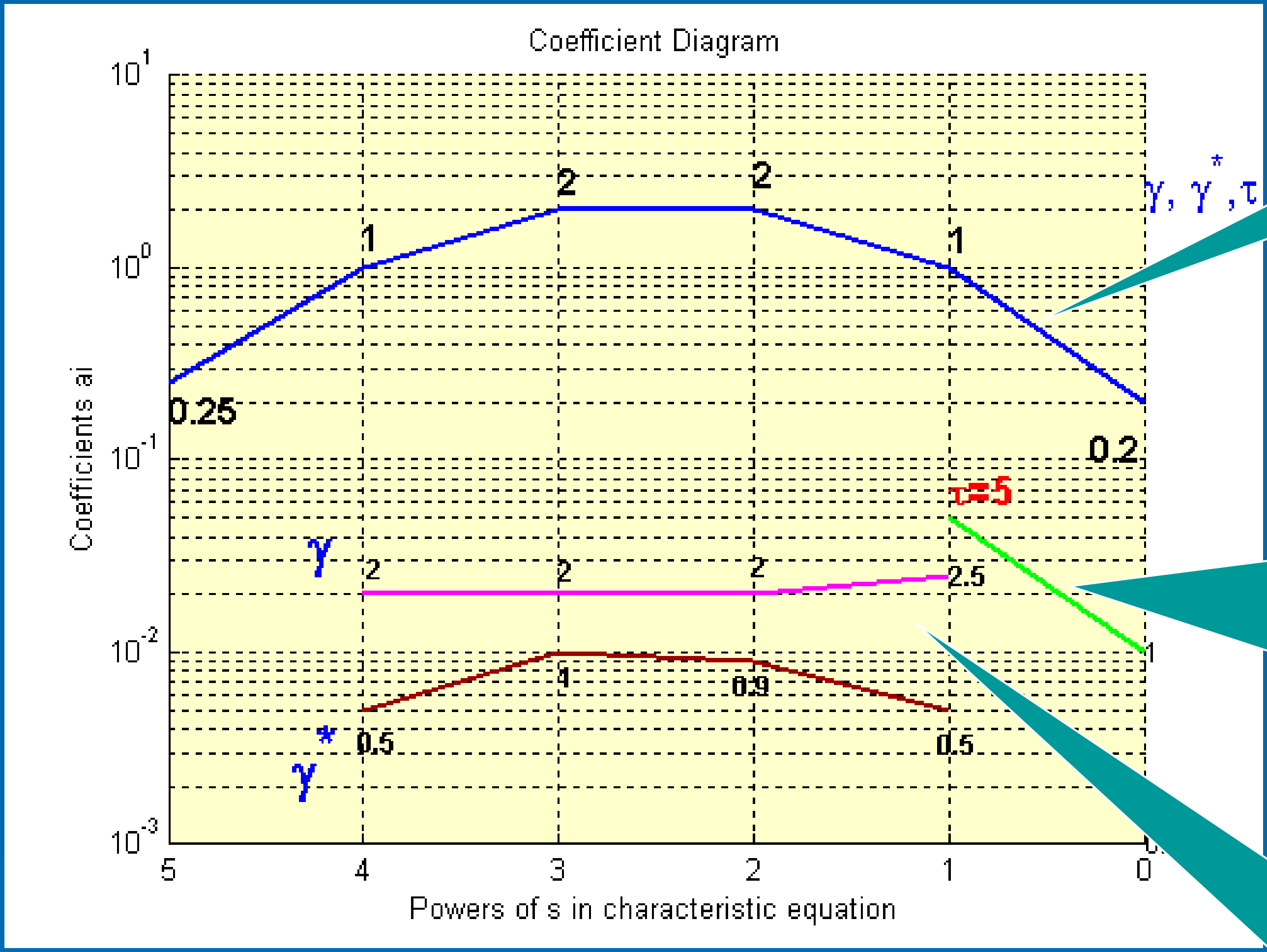
- The sufficient condition for instability is given as

$$\gamma_{i+1}\gamma_i \leq 1 \text{ for some } i = 1, \ldots n-2$$

Coefficient Diagram

A single semi log diagram that depicts following features of a system

- Response speed
- Stability
- Robustness



Coefficient curve ,more curvature more stable

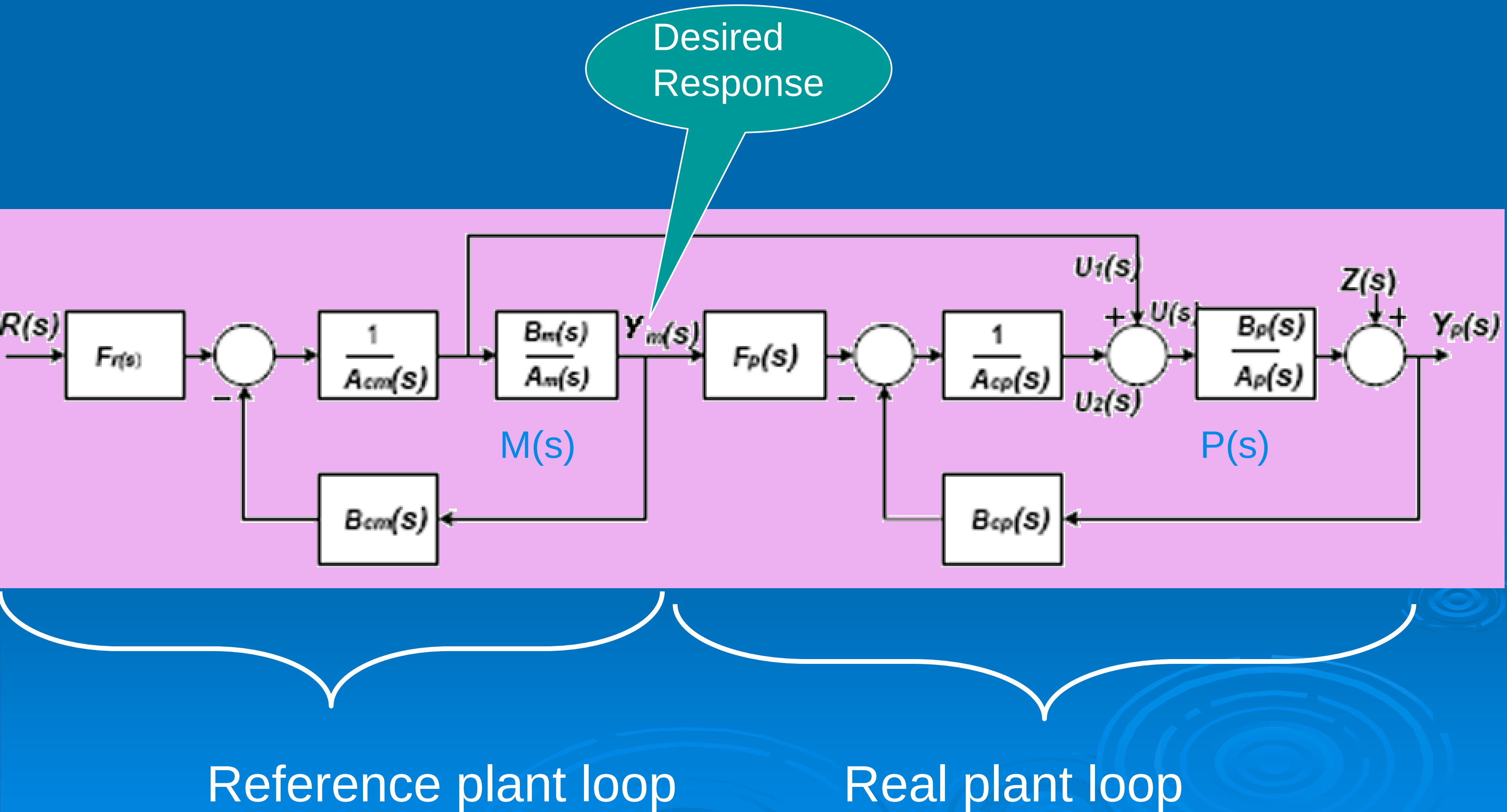
If the curve gets right end down the equivalent time constant decreases, response speed increases

The variation of the shape of the coefficient curve and the curves for stability indices and limits is a measure of Robustness

Coefficient Diagram for the characteristic polynomial

$$P(s) = 0.25s^5 + s^4 + 2s^3 + 2s^2 + s + 0.2$$

Proposed Control Structure



Proposed Control Strategy

- The two loop control structure is modified according to the CDM control structure.
- A reference plant is chosen such that the perturbation of the reference plant is large relative to the known nominal plant to be controlled.

Proposed Control Strategy

- The controller for the reference plant is designed using CDM
- It is shown that if the following condition is satisfied the real plant output tracks the reference plant output

$$\left(\frac{\Delta(s)}{1+\Delta(s)}\right) A_{cp}(s) = M(s)\left(B_{cp}(s) - F_p(s)\right)$$

- $\Delta(s)$ is the multiplicative perturbation of the reference plant relative to nominal plant model

Proposed Control Strategy

- The controller for an interval plant model of the actual plant is designed using the following

$$\frac{\Delta(s)}{1+\Delta(s)} \simeq 1 \qquad A_{cp}(s) \simeq M(s)\left(B_{cp}(s) - F_p(s)\right)$$

- $\Delta(s)$ is the multiplicative perturbation of the reference plant relative to nominal plant model

Proposed Control Strategy

- Even if the parameters of the actual plant deviate from the nominal plant parameter values $\frac{\Delta(s)}{1+\Delta(s)} \simeq 1$ still holds good.

- The controller designed gives robust performance for wide range of parameter variations in the actual plant around the nominal values.

Design Example 1

- Real plant

$$P(s) = \frac{K_m}{s(T_m s + 1)}, \quad K_m \in [12, 25], T_m \in [0.05\ , 0.5]$$

- Nominal plant

$$P_{nom}(s) = \frac{K_m}{sA_p'(s)} = \frac{18.3}{s(0.1s + 1)}$$

- Reference plant chosen

$$M(s) = \frac{1}{s(0.1s + 1)}$$

Results Example - 1

- Controller Parameters – Reference Plant

$$\frac{B_{cm}(s)}{A_{cm}(s)} = \frac{0.2720s + 1}{0.0512s + 0.128}$$

$$F_r(s) = 1$$

- Feedback Controller – Actual Plant

$$B_{cp}(s) = 0.272s + 1$$

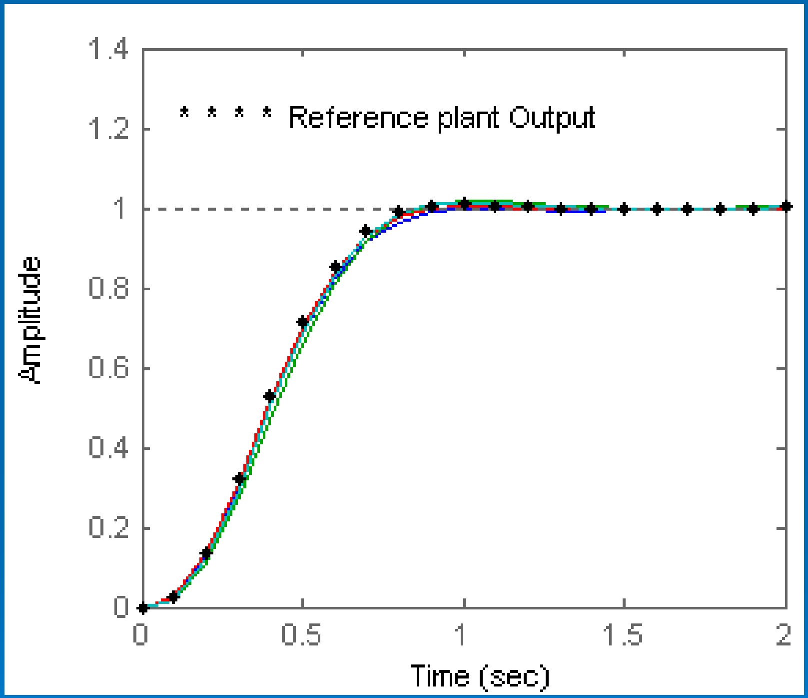
- Cascade Controller – Actual Plant

$$\frac{1}{A_{cp}(s)} = 0.36764s + 3.6764$$

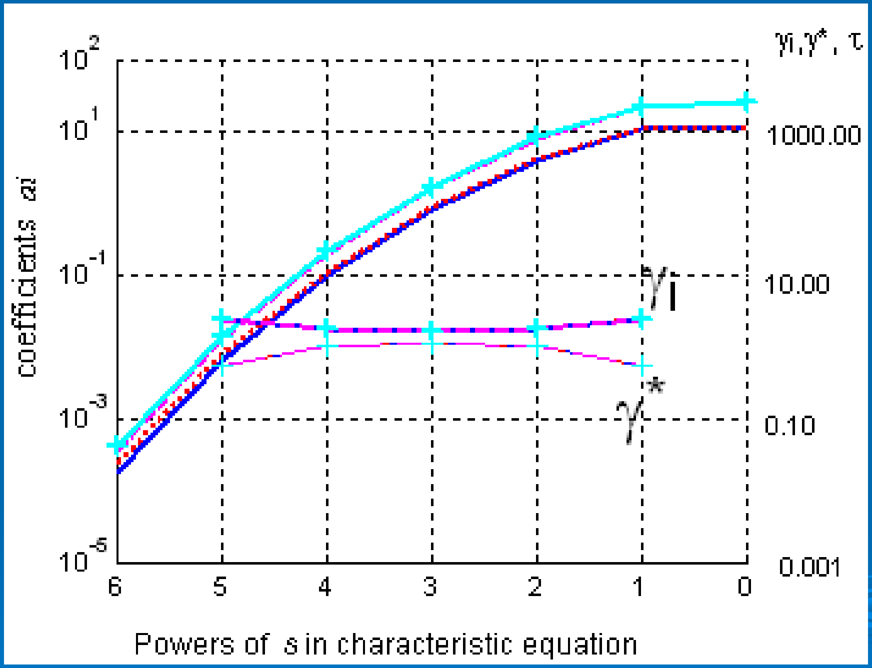
- Can be realized using PD Controllers

Simulation Results
Example 1
Performance Analysis

Step Response



Coefficient Diagram

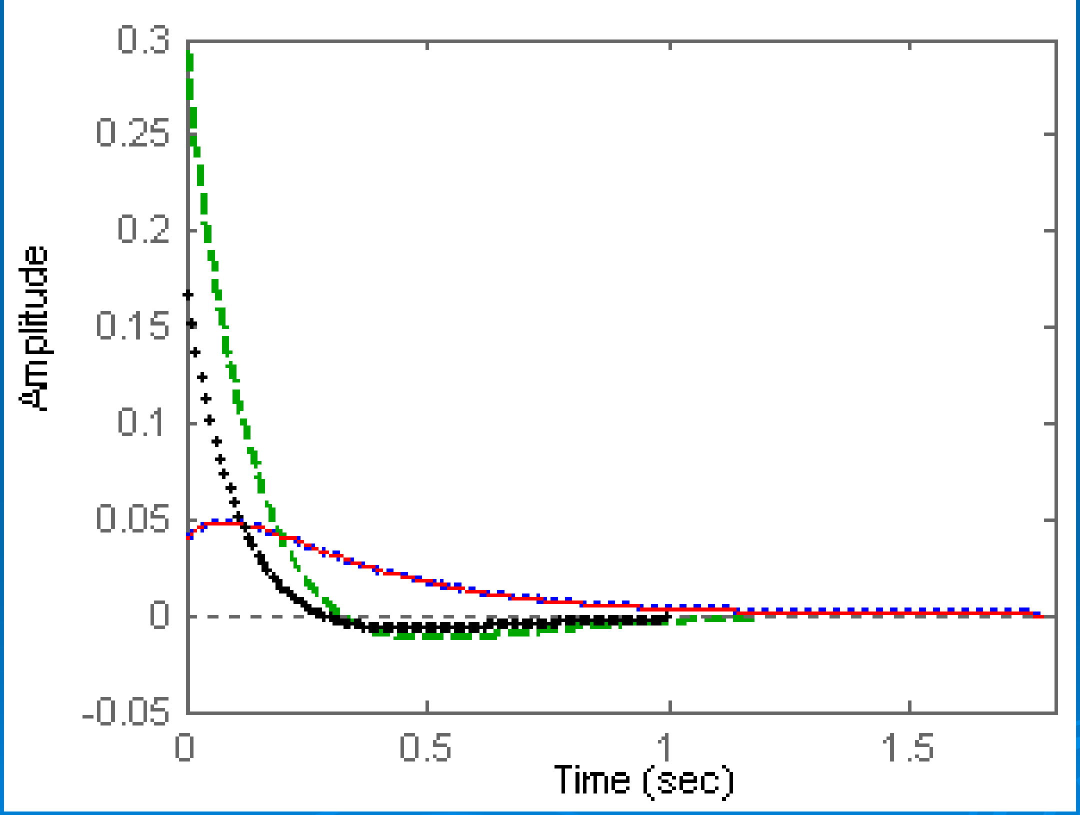


Inferences: 1. The Step responses exhibit excellent Model following or Tracking

2. Less variation in the shape of curves of Coefficient Diagram is an indicative of Robustness

Simulation Results
Example 1

Effect of unit step disturbance input



Inference: The Controller design offers rejection to step disturbance inputs

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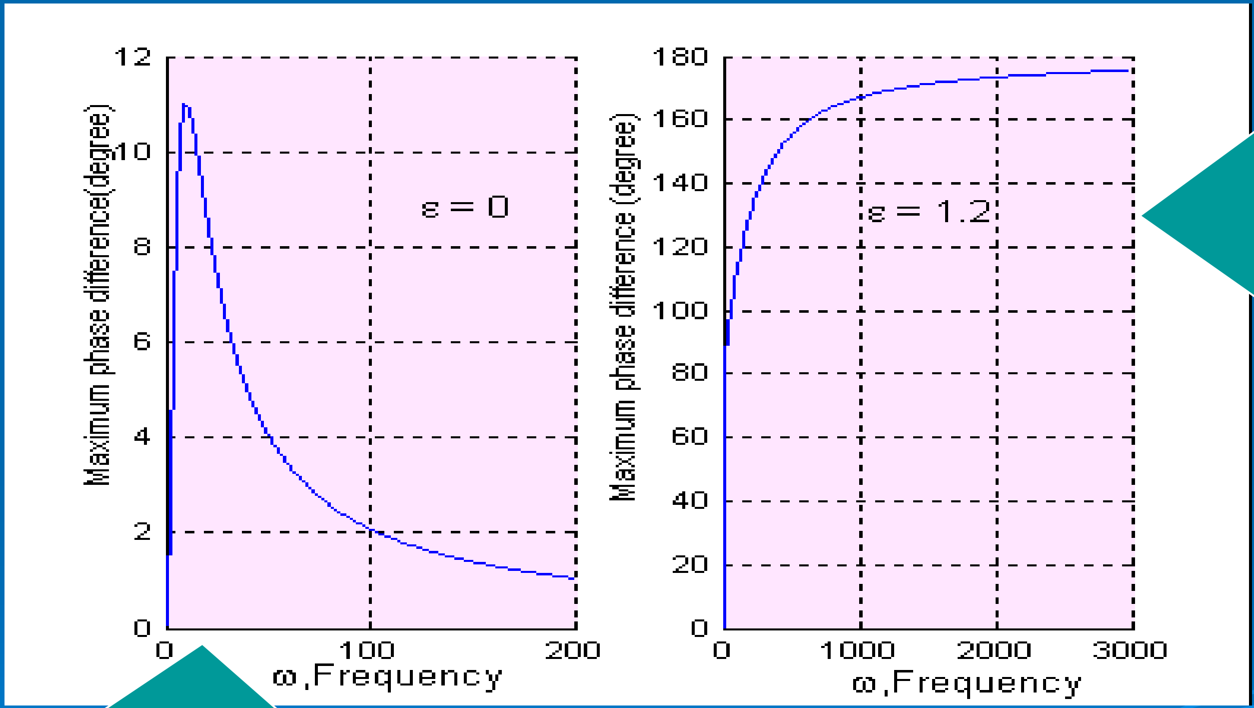
Simulation Results
Example 1

Robust Stability: Analyzed by Bounded phase Theorem

ε is the maximum possible excursion of plant parameters



Inference 2: Parameter perturbation can be increased to a value around 1.2 before the system becomes unstable

Inference 1: The maximum phase difference is around 11 degree (less than 180 degree) and indicates that the controllers designed robustly stabilize the given interval plant

Design Example 2

- Real plant

$$P(s) = \frac{K_m}{s(T_m s+1)(T_f s+1)}, K_m \in [0.5, 1.5], T_m = [0.5, 2], T_f = [0.1, 0.5]$$

- Nominal Plant

$$P_{nom}(s) = \frac{K_m}{sA'_p(s)} = \frac{1}{s(0.25s+1)(s+1)}$$

- Reference Plant

$$M(s) = \frac{0.1}{s(0.25s+1)(s+1)}$$

Results Example - 2

- Controller Parameters – Reference Plant

$$\frac{B_{cm}(s)}{A_{cm}(s)} = \frac{0.4733s^2 + 6.4198s + 10}{0.0082s^2 + 0.2551s + 1.358}$$

$$F_r(s) = 10$$

- Feedback Controller – Real Plant

$$B_{cp} = 0.1s^2 + 6.4198s + 10$$

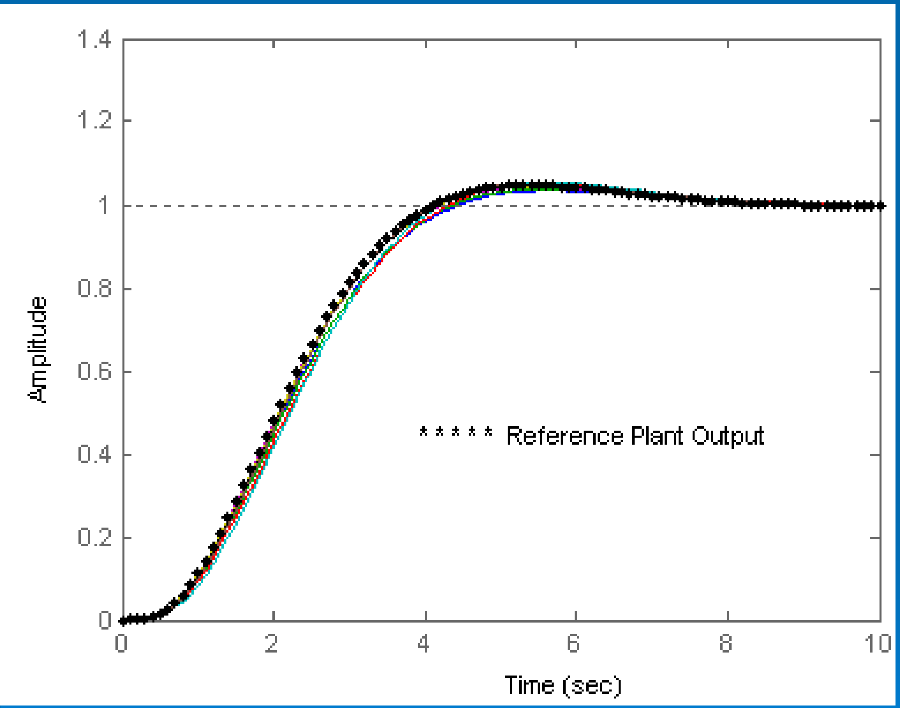
- Cascade Controller – Real Plant

$$\frac{1}{A_{cp}} = \frac{(2.5s + 10)(s + 1)}{0.1s + 6.4198}$$

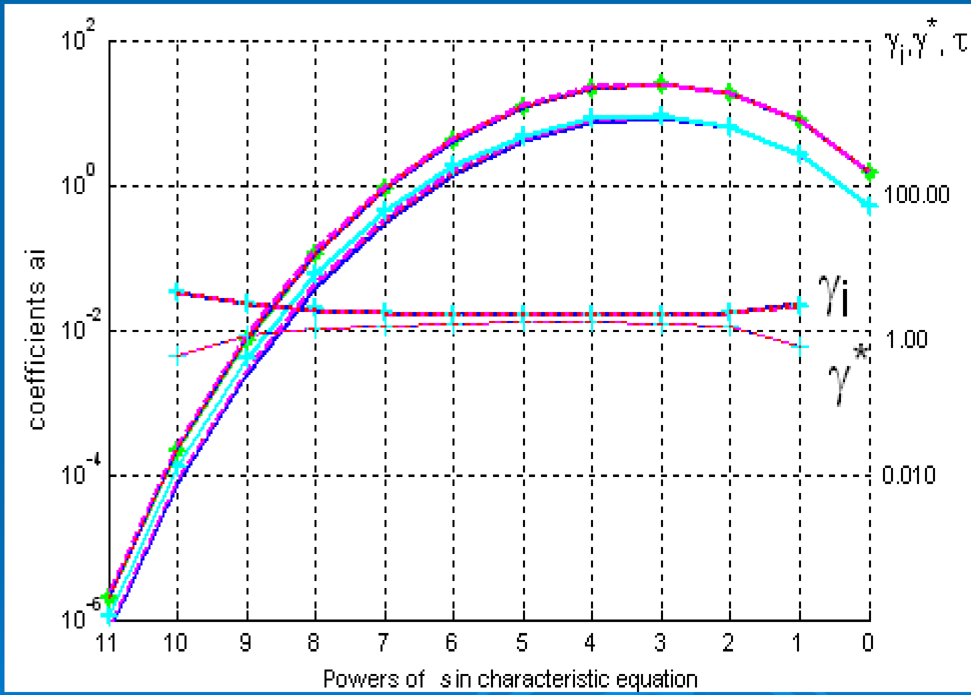
- Can be realized using PD Controllers and Derivative filters

Simulation Results
Example 2
Performance Analysis

Step Response



Coefficient Diagram



Inference: The step responses and the coefficient diagram exhibit good tracking and robust design features

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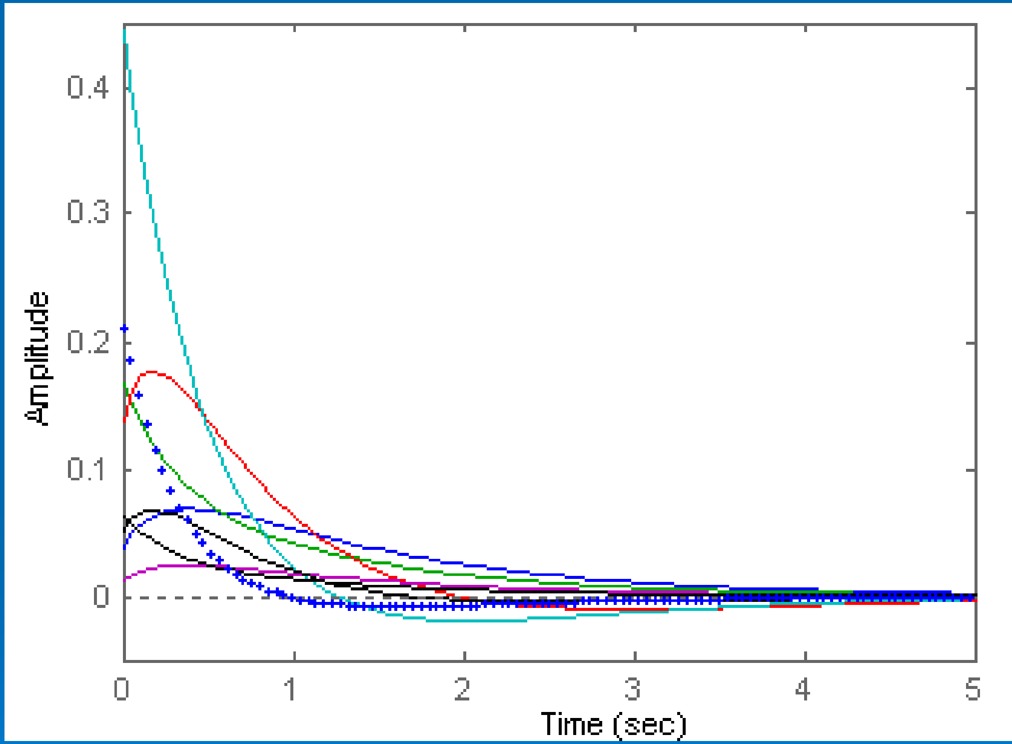
Simulation Results
Example 2

Effect of unit step disturbance input



Inference: The responses depict that the design offers rejection to step disturbance inputs

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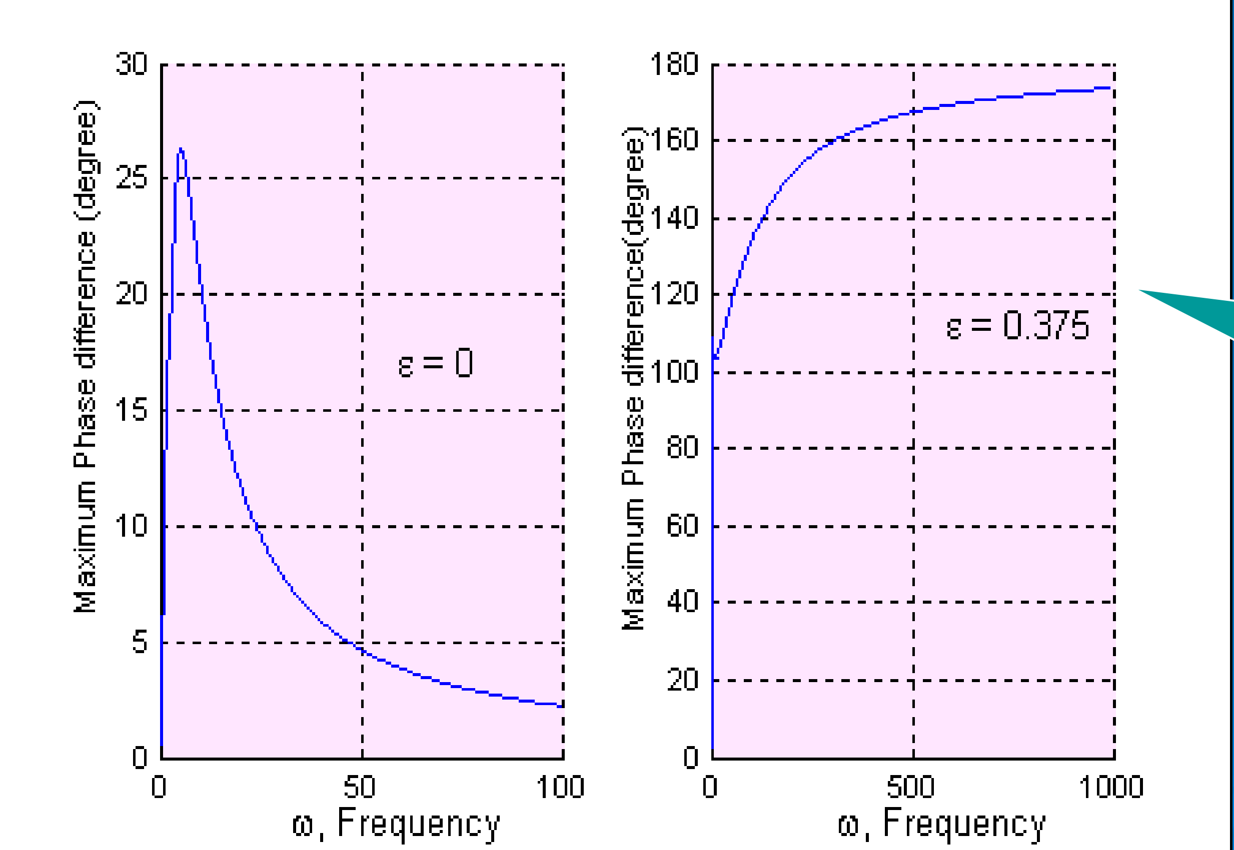
Simulation Results Example 2

Robust Stability

- Analyzed by Bounded phase Theorem



The parameter perturbation can be increased to a value around 0.375 before the system becomes unstable

Conclusions

- A novel Control strategy is proposed by modifying the two loop control structure according to CDM
- The mathematical relationships that contribute to the robustness have been derived
- Two examples of interval plants are considered to explain the strategy

Conclusions

- In each case the following characteristics are analyzed
  1. Performance
  2. Disturbance rejection
  3. Robust stability

- Bounded Phase condition of Robust control has been applied to analyze robust stability

Conclusions

- Simulation results of both the examples agree with the mathematical conditions derived
- The design offers robustness to the actual plant in the presence of wide parameter variations

Thank You