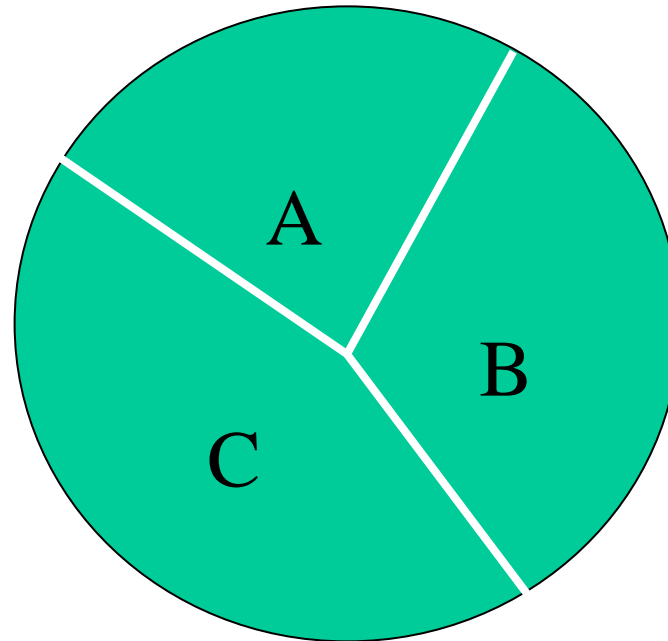# A New Risk Definition for Uncertain Portfolio Selection

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What is portfolio selection?

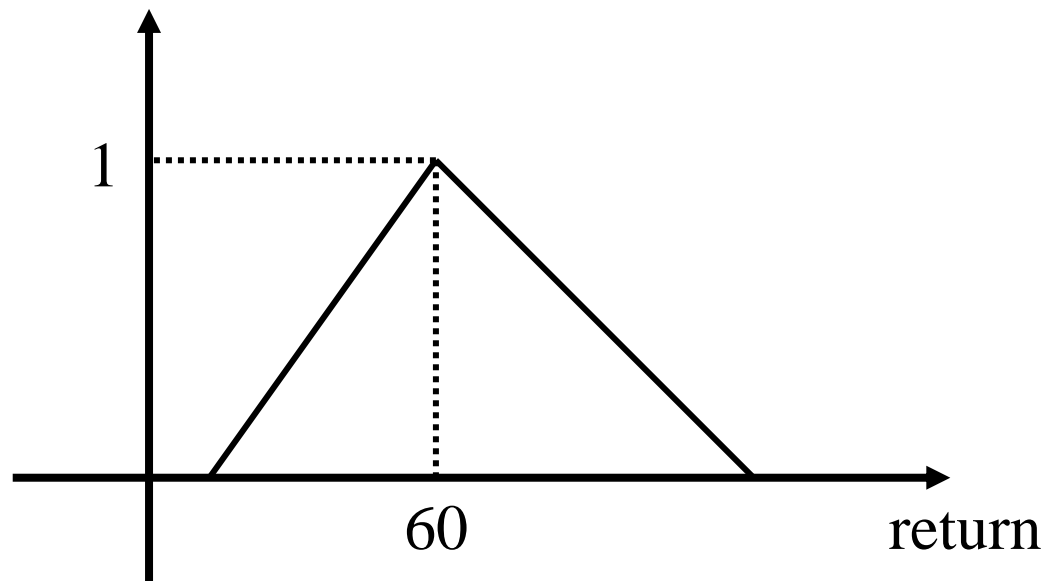To determine capital allocation proportions among securities.

How to represent a return?

1933: Kolmogroff: Random Variable

1965: Zadeh: Fuzzy Variable

2007: Liu: Uncertain Variable

Return is fuzzy?

Membership function of return

What is risk for uncertain portfolio investment?

Variance?

Consider returns of two portfolios A and B:

A: (0,1) with mean 0.5.

B: (100,101) with mean 100.5.

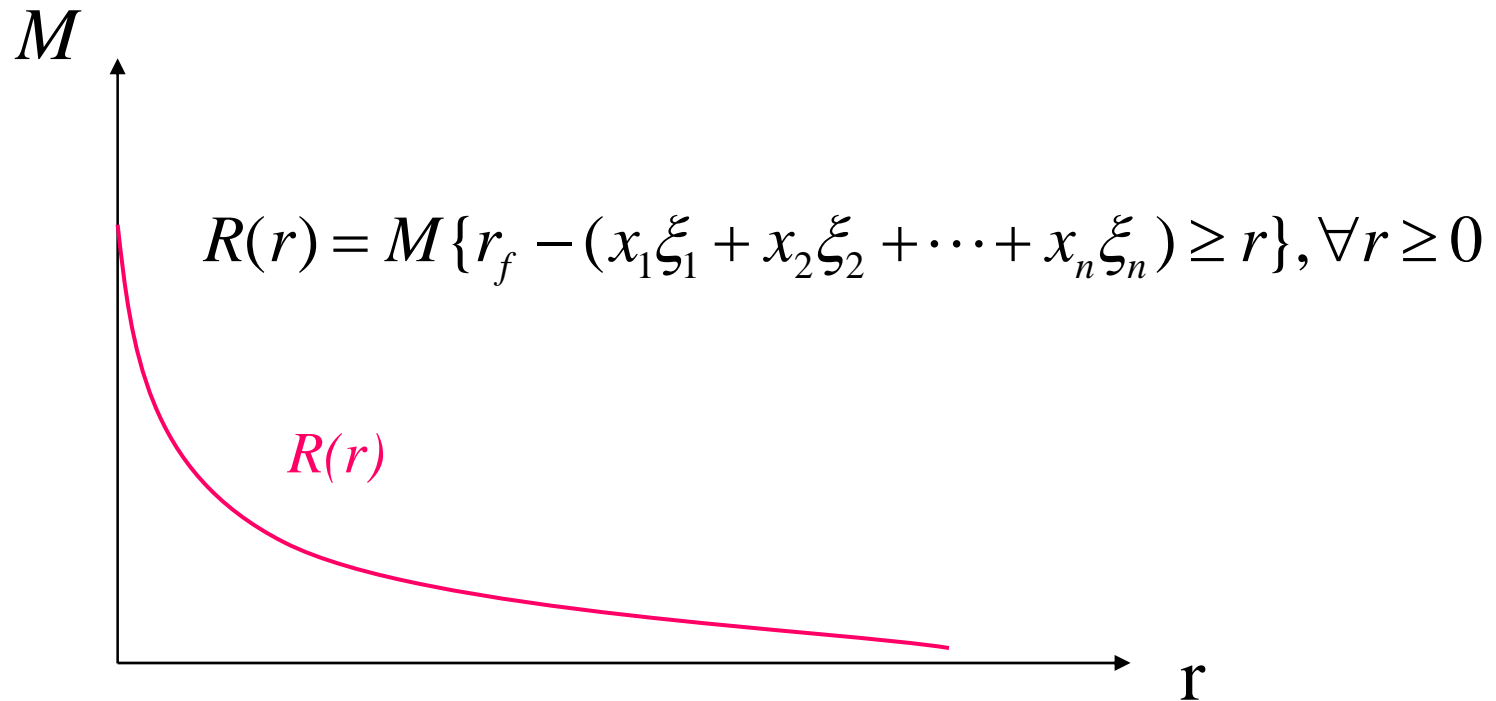Do you think portfolio A is as same risky as portfolio B?

What is risk curve?

$$\text{Loss}: 0 - (x_1 \xi_1 + x_2 \xi_2 + \cdots + x_n \xi_n) \geq 0,$$

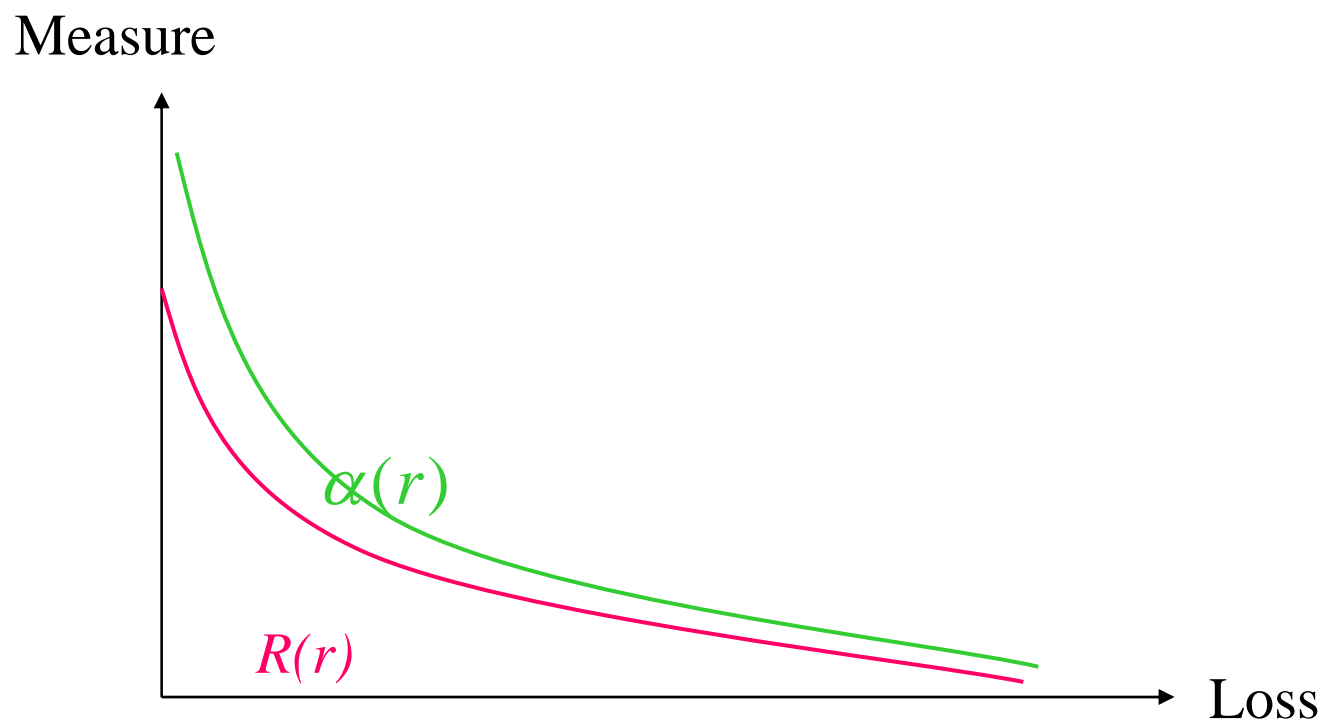$$\text{Loss}: r_f - (x_1 \xi_1 + x_2 \xi_2 + \cdots + x_n \xi_n) \geq 0,$$

$$\text{Risk curve}: R(r) = M\{r_f - (x_1 \xi_1 + x_2 \xi_2 + \cdots + x_n \xi_n) \geq r\}, \forall r \geq 0.$$

Intuitive; Panoramic!

Risk curve:

$$R(r) = M\{r_f - (x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n) \geq r\}, \forall r \geq 0$$

M

R(r)

r

Risk curve and safe portfolio

A portfolio is safe if $R(x_1, x_2, \cdots, x_n; r) \leq \alpha(x_1, x_2, \cdots, x_n; r)$.

Mean-Risk Model for Random Portfolio Selection
(Huang, EJOR, 2008)

$$
\begin{cases}
\max E[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] \\
\text{subject to:} \\
\quad \Pr\{r_f - (x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n) \geq r\} \leq \alpha(r), \ \forall r \geq 0 \\
\quad x_1 + x_2 + \cdots + x_n = 1 \\
\quad x_i \geq 0, \ i = 1, 2, \cdots, n.
\end{cases}
$$

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Mean-Risk Model for Uncertain Portfolio Selection

$$
\begin{cases}
\max E[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] \\[2mm]
\text{subject to:} \\[2mm]
\quad M\{r_f - (x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n) \geq r\} \leq \alpha(r), \forall r \geq 0 \\[2mm]
\quad x_1 + x_2 + \cdots + x_n = 1 \\[2mm]
\quad x_i \geq 0, i = 1, 2, \cdots, n
\end{cases}
$$

where M is the uncertain measure which satisfies the normality, monotonicity, self-duality and countable subadivity axioms.

The idea of *risk curve* is also applicable to structural reliability and other areas.