A New Risk Definition for Uncertain Portfolio Selection

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What is portfolio selection?

To determine capital allocation proportions among securities.
How to represent a return?

1933: Kolmogroff: Random Variable

1965: Zadeh: Fuzzy Variable

2007: Liu: Uncertain Variable
Return is fuzzy?

Membership function of return
What is risk for uncertain portfolio investment?

Variance?

Consider returns of two portfolios A and B:

A: (0,1) with mean 0.5.

B: (100,101) with mean 100.5.

Do you think portfolio A is as same risky as portfolio B?
What is risk curve?

\[
\text{Loss} : 0 - (x_1 \xi_1 + x_2 \xi_2 + \cdots + x_n \xi_n) \geq 0, \]

\[
\text{Loss} : r_f - (x_1 \xi_1 + x_2 \xi_2 + \cdots + x_n \xi_n) \geq 0, \]

\[
\text{Risk curve} : R(r) = M \{ r_f - (x_1 \xi_1 + x_2 \xi_2 + \cdots + x_n \xi_n) \geq r \}, \forall r \geq 0. \]

Intuitive; Panoramic!
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Risk curve:

\[ R(r) = M \{ r_f - (x_1 \xi_1 + x_2 \xi_2 + \cdots + x_n \xi_n) \geq r \}, \forall r \geq 0 \]
A portfolio is safe if \( R(x_1, x_2, \cdots, x_n; r) \leq \alpha(x_1, x_2, \cdots, x_n; r) \).
Mean-Risk Model for Random Portfolio Selection
(Huang, EJOR, 2008)

\[
\begin{aligned}
\max & \ E[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] \\
\text{subject to:} & \\
\Pr\{r_f - (x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n) \geq r\} & \leq \alpha(r), \ \forall r \geq 0 \\
x_1 + x_2 + \cdots + x_n & = 1 \\
x_i & \geq 0, \ i = 1, 2, \cdots, n.
\end{aligned}
\]
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Mean-Risk Model for Uncertain Portfolio Selection

\[
\begin{align*}
\max & \ E[x_1 \xi_1 + x_2 \xi_2 + \cdots + x_n \xi_n] \\
\text{subject to:} & \\
& M\{r_f - (x_1 \xi_1 + x_2 \xi_2 + \cdots + x_n \xi_n) \geq r\} \leq \alpha(r), \ \forall r \geq 0 \\
& x_1 + x_2 + \cdots + x_n = 1 \\
& x_i \geq 0, \ i = 1, 2, \cdots, n
\end{align*}
\]

where \(M\) is the uncertain measure which satisfies the normality, monotonicity, self-duality and countable subaditivitity axioms.
The idea of *risk curve* is also applicable to structural reliability and other areas.