Uncertainty Theory

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Some information may be represented by human language like

"about 100km", "approximately 39 °C", "roughly 80kg"

"low speed", "middle age", "big size"

How do we understand them?

How do we model them?

Some scholars invented some concepts like

Randomness, Fuzziness, Roughness, Vagueness, Greyness, Uncertainty

What is the starting point?

Fortunately, some measures were invented by axiomatic methods!

1933: Probability Measure (A.N. Kolmogoroff)
1978: Possibility Measure (L.A. Zadeh)
2007: Uncertain Measure (B. Liu)

"about 100km" is randomness?

L.J. Savage said:

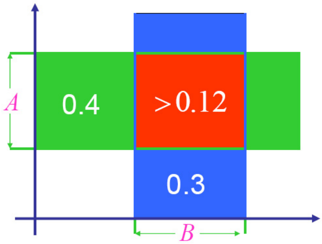
A rational man behaves as if he used subjective probabilities.

If so, it is subjective probability!

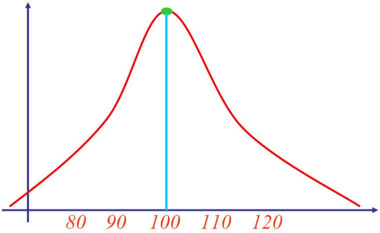
J. Cohen Said:

The compound event's subjective probability is usually too high.

If so, it is not subjective probability!



"about 100km" is a fuzzy concept?



The distance is "exactly 100km" (neither more nor less) with possibility 1.

I do not believe! Do you?

"about 100km" is not a fuzzy concept
because it cannot be quantified by Zadeh's possibility measure!

Uncertain Theory

Uncertain Measure (Liu, UT-2, 2007)

Axiom 1. (Normality) $\mathcal{M}\{\Gamma\} = 1$ for the universal set.

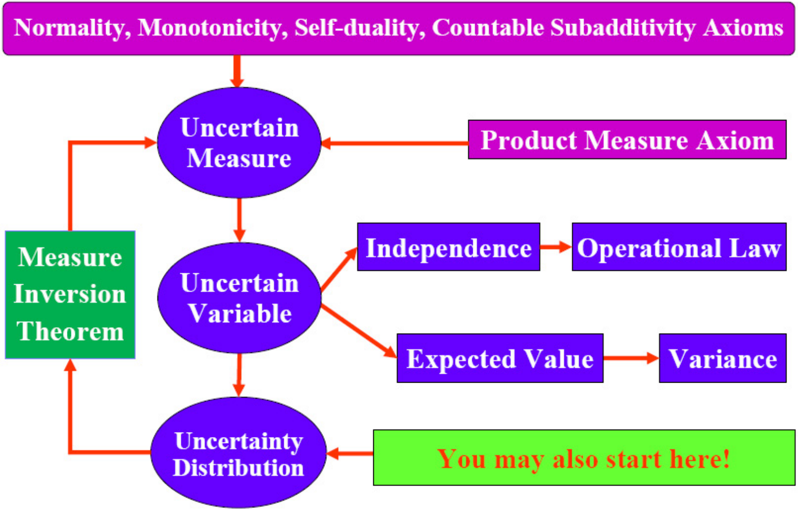
Axiom 2. (Monotonicity) $\mathcal{M}\{\Lambda_1\} \le \mathcal{M}\{\Lambda_2\}$ whenever $\Lambda_1 \subset \Lambda_2$.

Axiom 3. (Self-Duality) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.

Axiom 4. (Countable Subadditivity) $\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \le \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}$.

Axiom 5. (Product Measure) $\mathcal{M}\left\{\prod_{k=1}^{n} \Lambda_k\right\} = \min_{1\le k\le n} \mathcal{M}_k\{\Lambda_k\}$.

Uncertainty Theory



Risk Index (Liu, JUS, 2010)

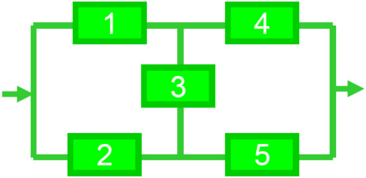
Assume

(1) a system contains uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$, *and*

(2) some specified loss occurs if and only if $L(\xi_1, \xi_2, \cdots, \xi_n) \le 0$.

Then the risk index is

$$Risk = \mathcal{M}\{L(\xi_1, \xi_2, \cdots, \xi_n) \le 0\}.$$



Risk Index Theorem (Liu, JUS, 2010)

Assume

(1) $\xi_1, \xi_2, \cdots, \xi_n$ *have uncertainty distributions* $\Phi_1, \Phi_2, \cdots, \Phi_n$,

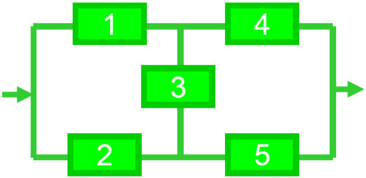
(2) the loss function L *is strictly increasing,*

(3) some specified loss occurs if and only if $L(\xi_1, \xi_2, \cdots, \xi_n) \leq 0$.

Then

$$\mathit{Risk} = \alpha$$

where α *is the root of* $L(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \cdots, \Phi_n^{-1}(\alpha)) = 0$.



$$L = (\xi_1 \wedge \xi_4) \vee (\xi_2 \wedge \xi_5) \vee (\xi_1 \wedge \xi_3 \wedge \xi_5) \vee (\xi_2 \wedge \xi_3 \wedge \xi_4) - T$$

Risk Index Theorem (Liu, JUS, 2010)

Assume

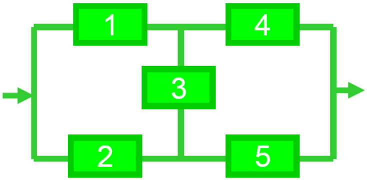
(1) $\xi_1, \xi_2, \cdots, \xi_n$ *are elements with reliabilities* $a_1, a_2, \cdots, a_n$,

(2) the truth function is f.

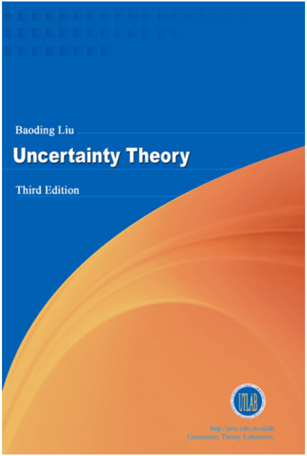
Then

$$Risk = \begin{cases} \sup\limits_{f(x_1,x_2,\cdots,x_n)=0} \min\limits_{1\le i\le n} \nu_i(x_i), & \text{if } \sup\limits_{f(x_1,x_2,\cdots,x_n)=0} \min\limits_{1\le i\le n} \nu_i(x_i) < 0.5 \\ 1 - \sup\limits_{f(x_1,x_2,\cdots,x_n)=1} \min\limits_{1\le i\le n} \nu_i(x_i), & \text{if } \sup\limits_{f(x_1,x_2,\cdots,x_n)=0} \min\limits_{1\le i\le n} \nu_i(x_i) \ge 0.5 \end{cases}$$

where x_i *take values either 0 or 1, and* $\nu_i(1) = a_i$ *and* $\nu_i(0) = 1 - a_i$.



$$f = (x_1 \wedge x_4) \vee (x_2 \wedge x_5) \vee (x_1 \wedge x_3 \wedge x_5) \vee (x_2 \wedge x_3 \wedge x_4)$$



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