Random Set Finite Element Method Application to Tunnelling

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CONTENT

- **Introduction**
  - Motivation
  - approaches in characterising uncertainty
- **Random set theory**
- **Random set finite element method (RS-FEM)**
- **Application to geotechnics**
- **Summary**

HOW TO DEAL WITH UNCERTAINTIES?

- **Common practice:**

  Parametric study based on experience

  Disadvantage:

  Worst case assumption not always obvious for complex, highly nonlinear systems
  Very cautious choice for parameters is common

- **Proposed approach by Peschl 2004 (Random Set-Finite Element Method):**

  Provide mathematical framework for dealing with uncertainties

  - without sacrificing complex analysis models (nonlinear FEM, BEM)
  - applicable for *ultimate limit state (failure)* AND *serviceability limit state (Deformations)*
  - without requiring substantial changes to finite element code
  - which appeals to practical engineers due to simplicity and usefulness

MATHEMATIZATION OF UNCERTAINTY

Interval

Interval arithmetic

Fuzzy sets

Imprecise, Vagueness, possibility theory

limited usability for reliability analysis

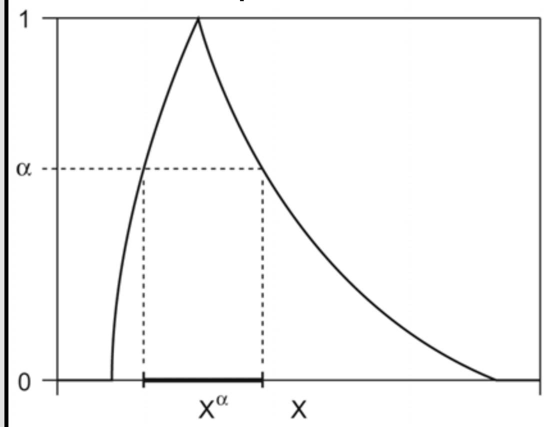
Precise probability

Probability theory,

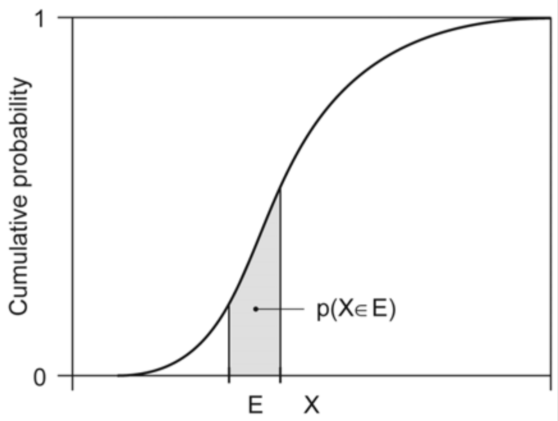
- Classical
- Frequentists
- Subjective (Bayes)

Simple Interval

Membership Function



Distribution Function



RANDOM SET THEORY:

✓ **Key problem:**

Combination of RANDOM (aleatory) and SET (epistemic) random set uncertainty => imprecise probability

✓ **Two traditions:**

- **Probability theory** (*aleatory uncertainty*)
- **Interval analysis** (*epistemic uncertainty*)

✓ **Combination of these traditions leads to *Random set model***

✓ **History**

- **Matheron G. (1975), Kendall DG. (1974) Theory of Random Sets**
- **Dempster, A.P. (1967) , Shafer, G. (1976) Theory of Evidence**
- **Tonon et al. (1996, 2000) Application of Random Sets in Rock Mechanics**
- **Schweiger & Peschl (2005) Random Set + Finite Element in Soil Mechanics**

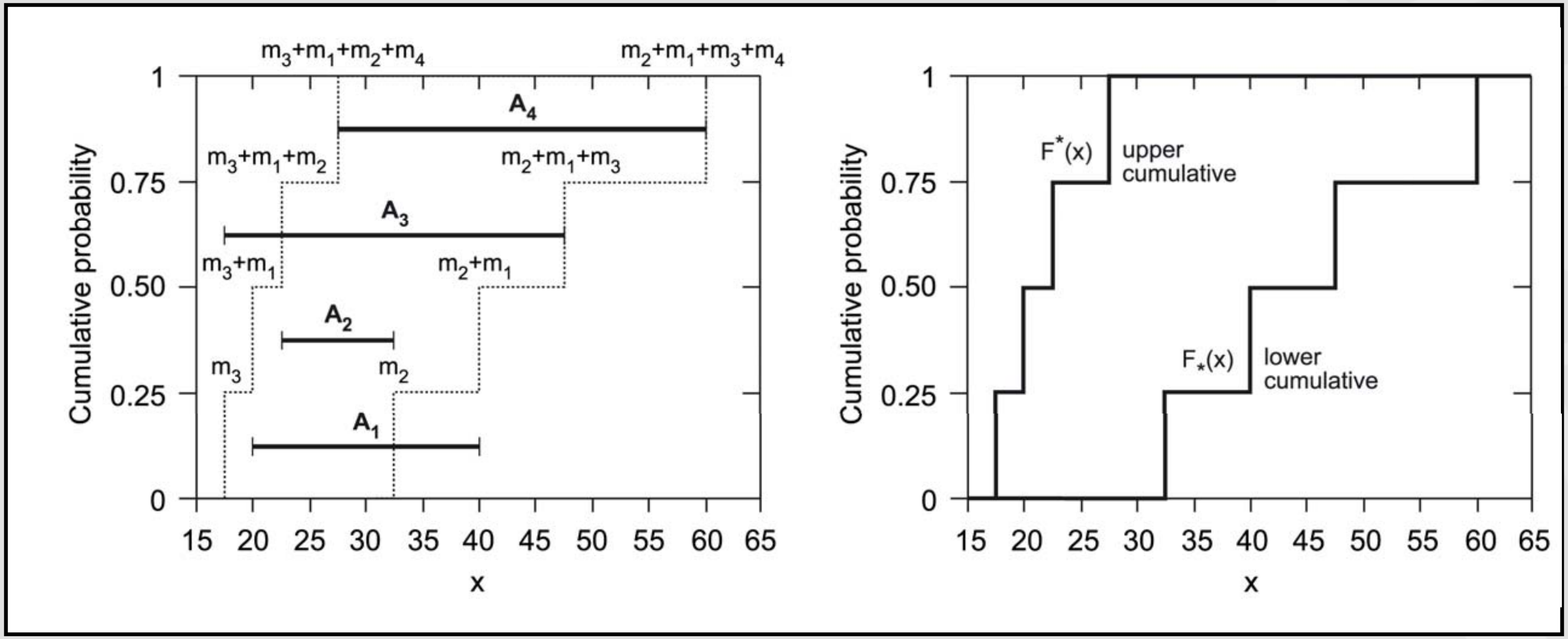
GENERAL REMARKS

- **Random set theory is a generalization of both interval analysis and probability theory**
- **Random set analysis gives the same answer as interval analysis, when only range information is available**
- **It gives the same answer as Monte-Carlo simulations, when information is abundant**
- **Random set theory can be interpreted as a generalization of probability theory were probabilities are assigned to sets (opposed to exclusive singletons)**
- **Accuracy depends primary on the information or data concerning the input parameters**

RANDOM SET THEORY:

- **Basic probability assignment *m*:**
  where n is the number of information sources

- $m = 1/n$ **as a default value (can be weighted)**



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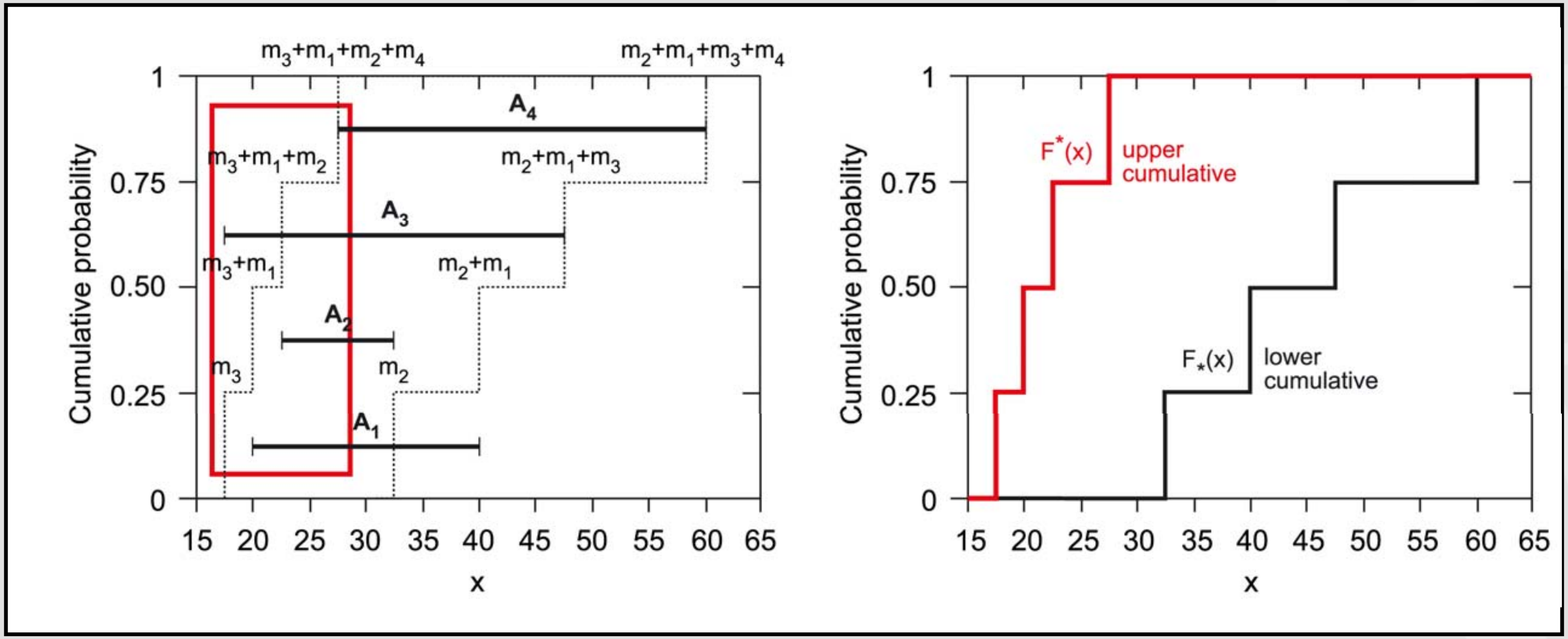

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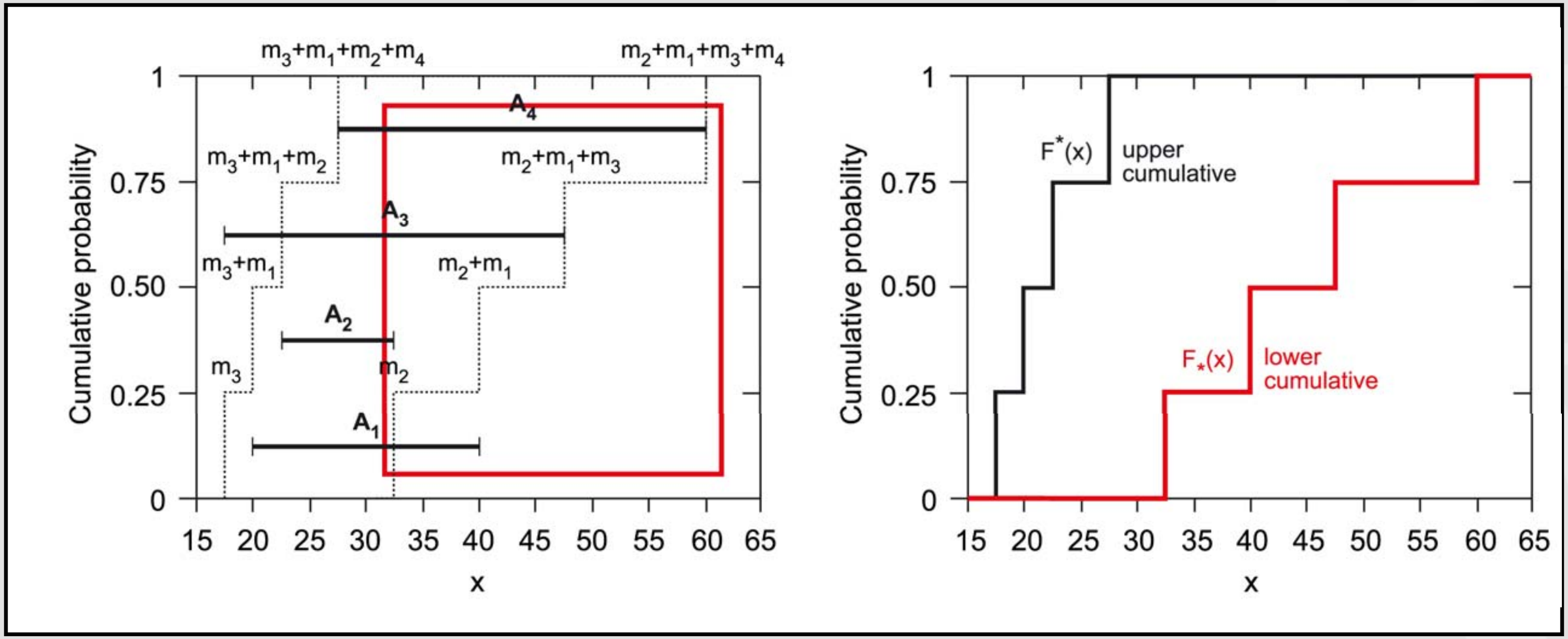
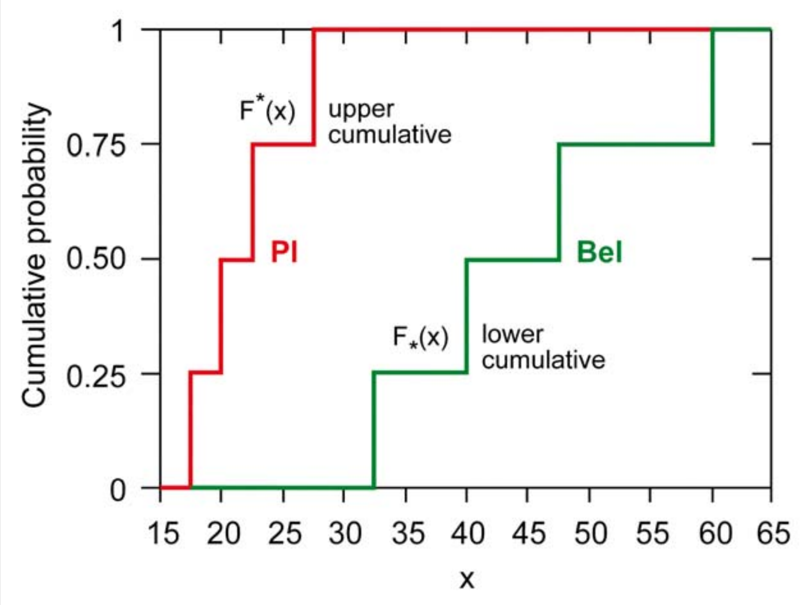
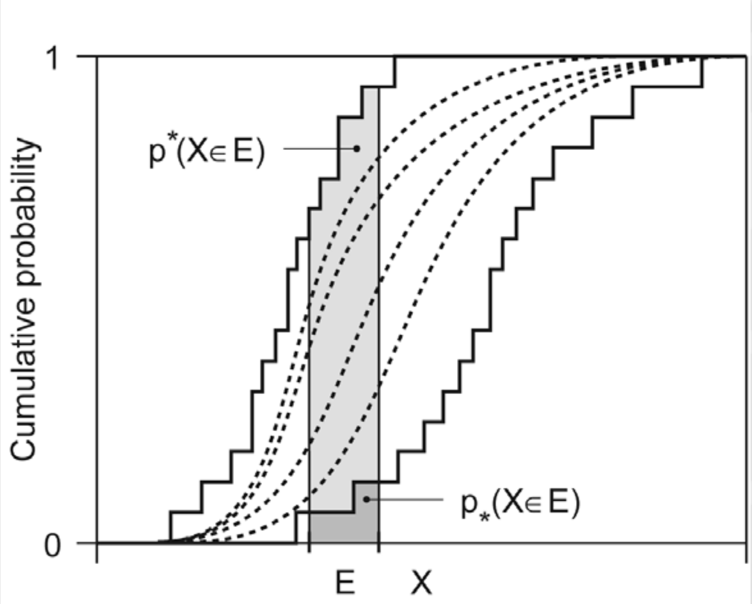


RANDOM SET THEORY:

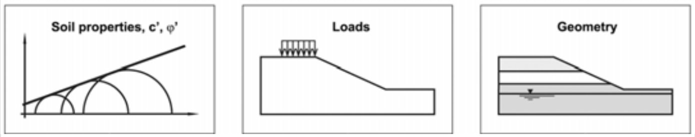
Plausibility function ***Pl*** (upper bound F^*)
Belief function ***Bel*** (lower bound F_*)

=> Interval bounds as CDF *envelope of all CDF's compatible with data*





Random Set Finite Element Method
Basic Procedure



- **Decision which parameters should be considered as basic variables**

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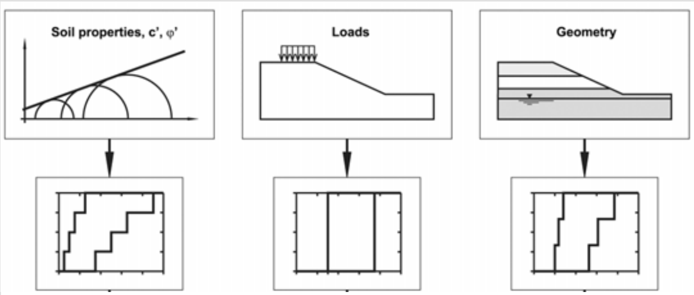

- **Decision which parameters should be considered as basic variables**

- **Construction of random sets**

Random Set Finite Element Method
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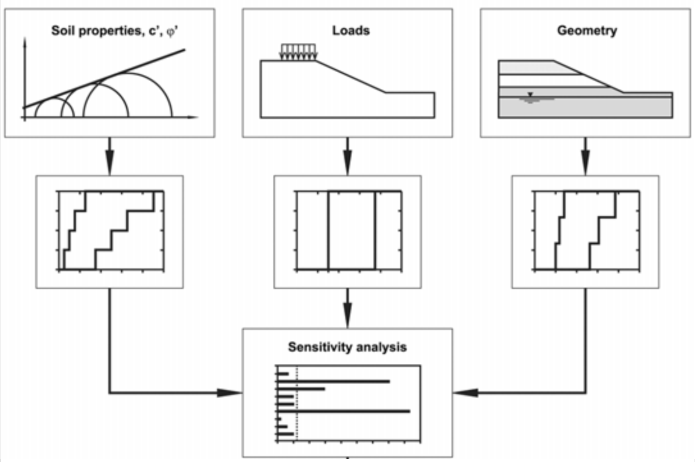

- Decision which parameters should be considered as basic variables
- Construction of random sets
- Sensitivity analysis to reduce the computational effort

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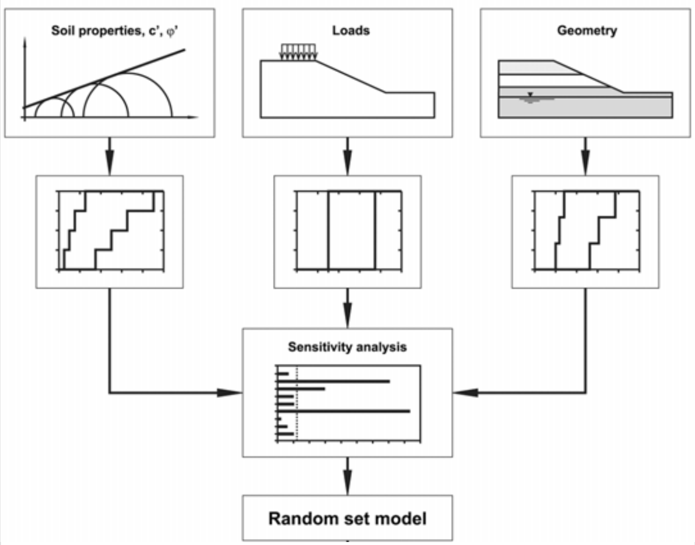

- **Decision which parameters should be considered as basic variables**
- **Construction of random sets**
- **Sensitivity analysis to reduce the computational effort**
- **Generate *calculation matrix* (random set model)**

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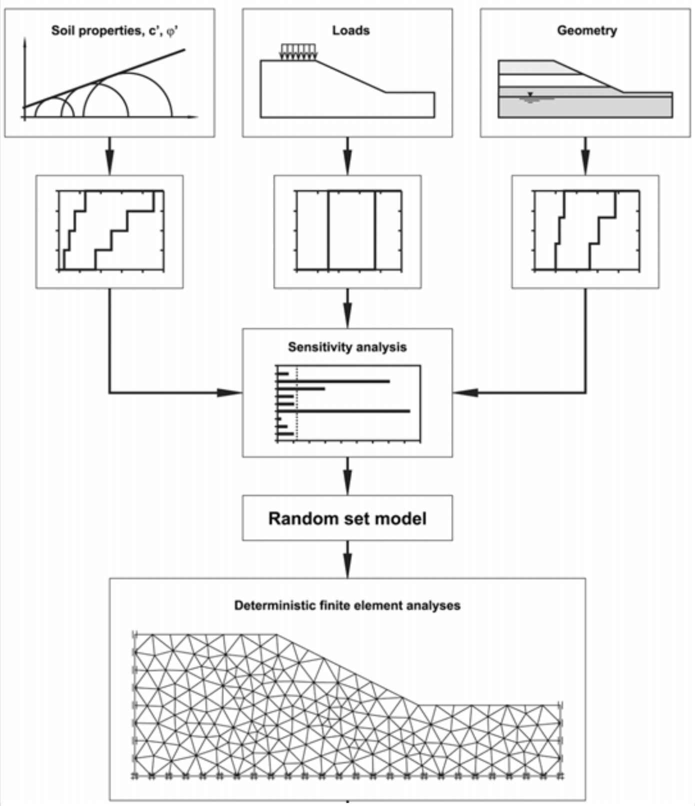

- Decision which parameters should be considered as basic variables
- Construction of random sets
- Sensitivity analysis to reduce the computational effort
- Generate *calculation matrix* (random set model)
- Perform all calculations

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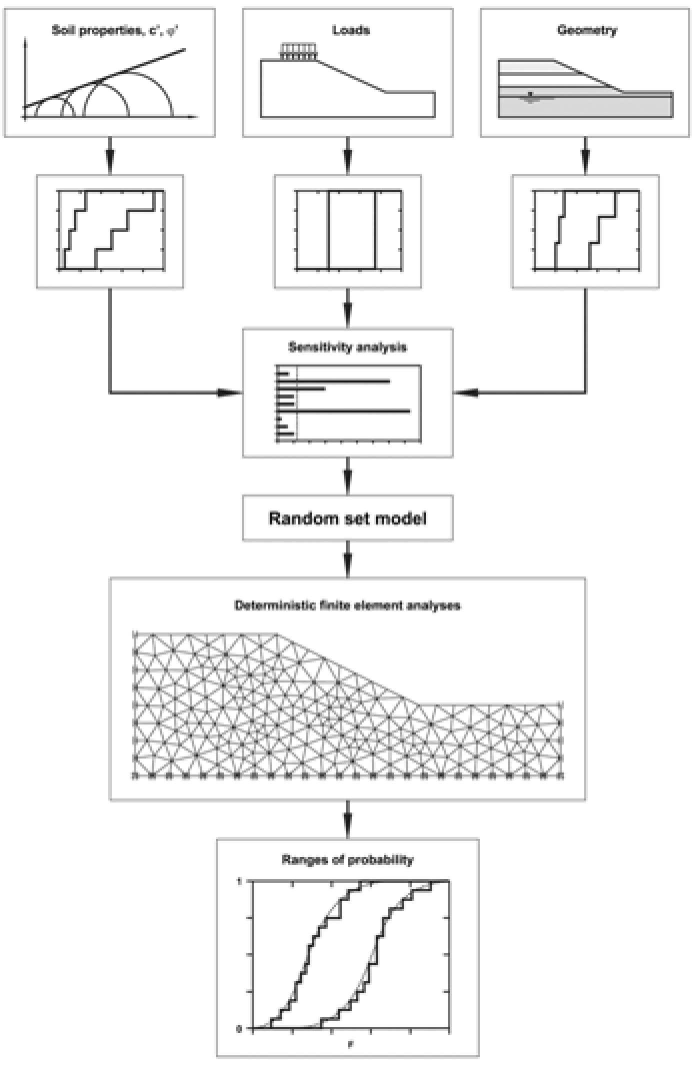
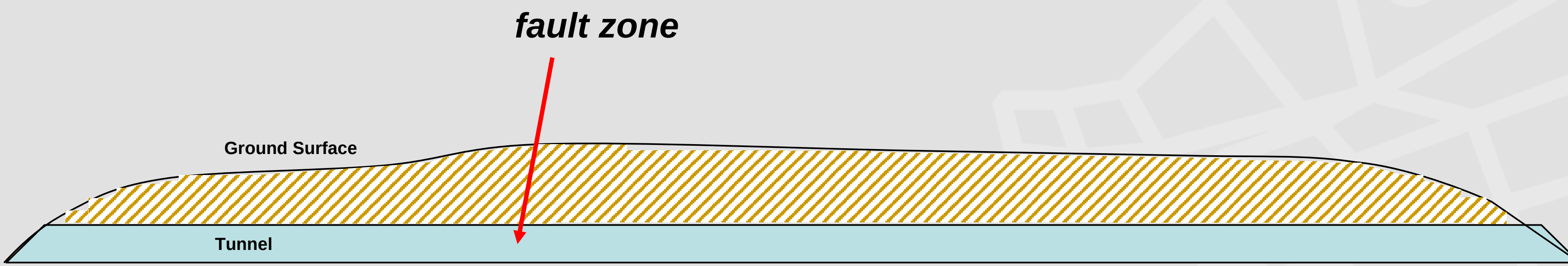


- Decision which parameters should be considered as basic variables
- Construction of random sets
- Sensitivity analysis to reduce the computational effort
- Generate *calculation matrix* (random set model)
- Perform all calculations
- Interval bounds as cumulative distributions

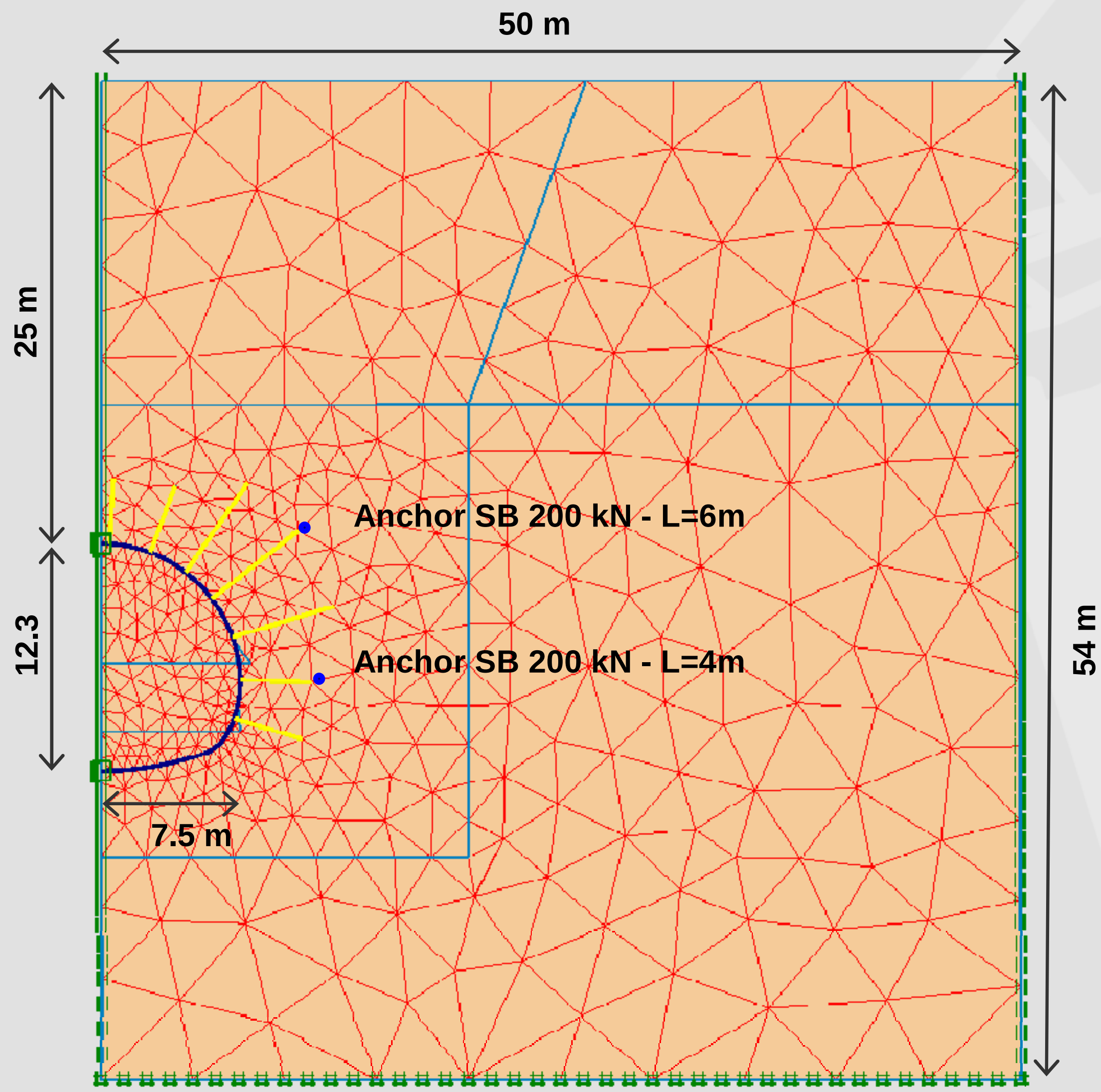
EXAMPLE TUNNEL EXCAVATION



	15m	100m	30m	290m	25m
Section	BQ1	BQ2	BQ3	BQ2	BQ1
Zone	HBI	HBII	HBIII	HBII	HBI

Zone	Unit	HBI	HBII	HBIII
E-Modul	MN/m²	75-175	500-800	85-200
φ	°	20-22	22-24	21-23
c	kN/m²	50-90	160-270	50-90
K_0 max	-	0.6	0.5	0.6
K_0 min	-	0.4	0.35	0.4
ν	-	0.35	0.27	0.35

BASIC INPUT VARIABLES



BASIC INPUT VARIABLES

Source 1: site investigation report — Hoek-Brown parameters of the rock mass

Parameter Description	Unit weight	Elastic modulus of intact rock	Rock mass rating	Geological strength index	Unconfined compressive strength of intact rock	HB parameter
Symbol	γ	E_i	RMR	GSI	σ_{ci}	m_i
Unit	kN/m³	GPa	-	-	MPa	-
Value/Range	24	19-25	20-30	30-40	10-50	15-25

Source 2: Previous Experience — in form of correlation relationships

✓ Hoek & Brown 1997

$$E_m = \sqrt{\frac{\sigma_{ci}}{100}}10^{(GSI-10)/40}$$

✓ Hoek et al. 2002

$$\varphi' = \sin^{-1}\left[\frac{6am_b(s+m_b\sigma'_{3n})^{a-1}}{2(1+a)(2+a)+6am_b(s+m_b\sigma'_{3n})^{a-1}}\right]$$

Source 3: Expert's opinion

Random Sets:

RSM	Probability	φ	E-Module (MN/m²)	Cohesion (kPa)	Relaxation Factor		
					Stage 1	Stage 2	Stage 3
Set 1	0.5	37-47	1300-2300	80-130	0.4-0.6	0.3-0.5	0.2-0.4
Set 2	0.5	53-60	1900-3400	160-280	0.3-0.5	0.2-0.4	0.1-0.3

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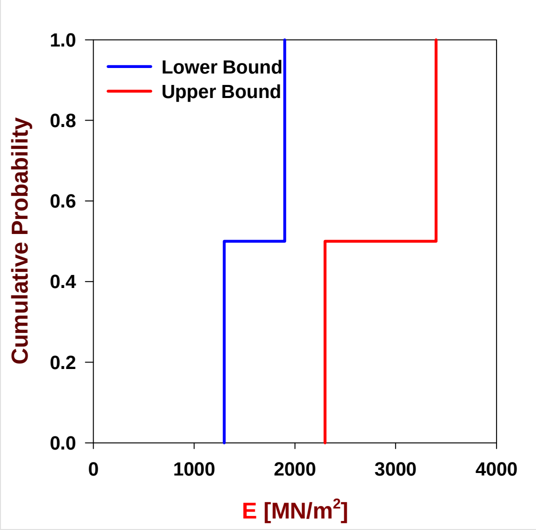

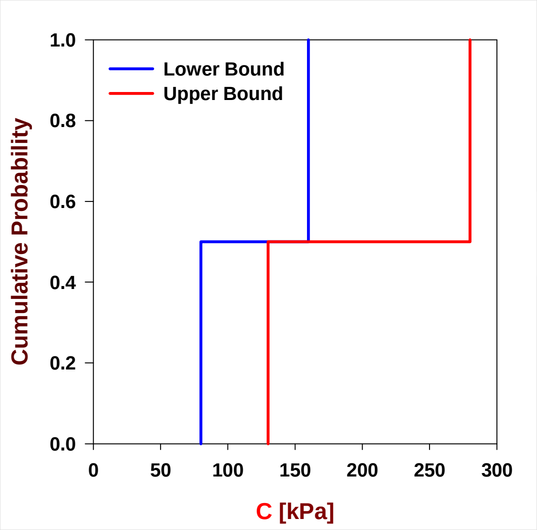

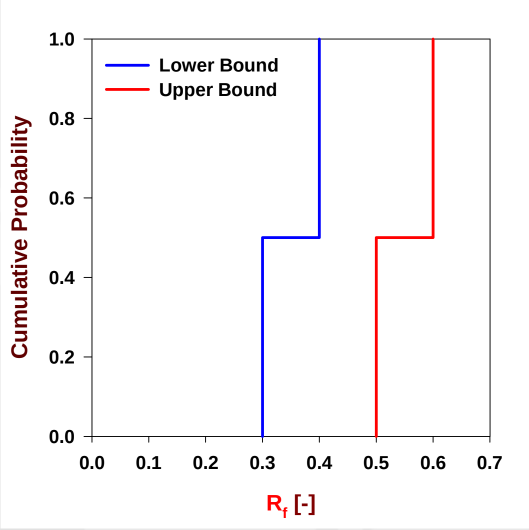

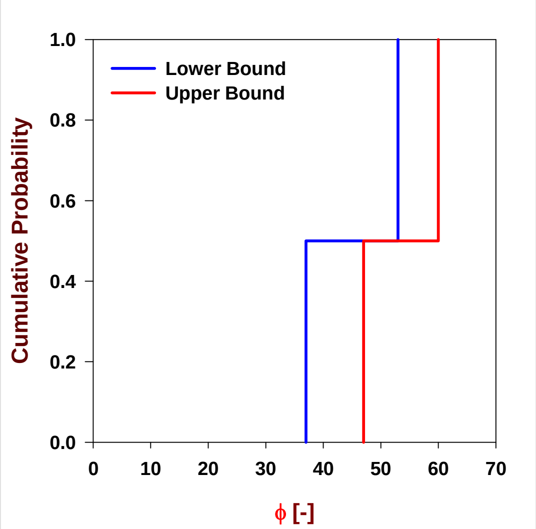

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RESULTS DERIVING PROCEDURE IN RS-FEM

Table of Input parameters in deterministic FE

Run	m(Ai)	Param	Set No.	LLLL	LLLU	LLUL	LLUU	--	UULL	UULU	UUUL	UUUU
1-16	0.50	E	1	1300	1300	1300	1300		2300	2300	2300	2300
	0.50	C	1	80	80	80	80		130	130	130	130
	0.50	φ	2	53	53	60	60		53	53	60	60
	0.50	Rf	2	0.3	0.5	0.3	0.5		0.3	0.5	0.3	0.5

Randomness

Corresponding Results: Moment

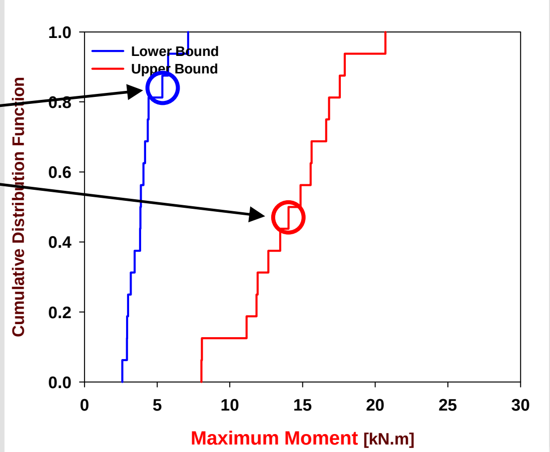
10.90	7.77	13.80	10.00		8.64	6.02	11.04	8.27

Max | Min

Probability measure

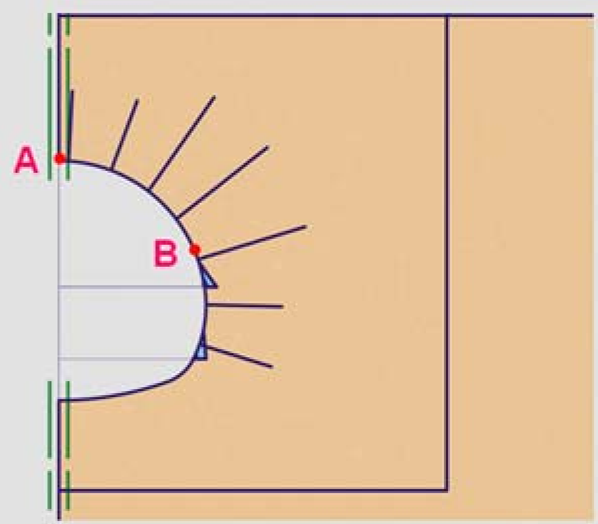
$$m(f(E_1,C_1,\Phi_2,R_2)) = m(E_1).m(C_1).m(\Phi_2).m(R_2)$$
$$= 0.5\times0.5\times0.5\times0.5$$
$$= 0.0625$$

6.02<Moment<13.80

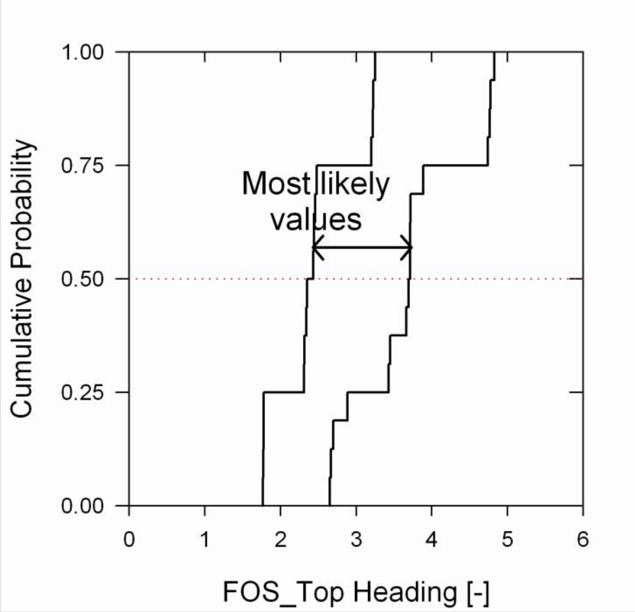


EVALUATED RESULTS

- **Vertical displacement of crown (Point A)**
- **Vertical and horizontal displacement of side wall (Point B)**
- **Maximum normal force in lining**
- **Maximum bending moment in lining**
- **Factor of Safety at stage 1 (before lining is placed)**



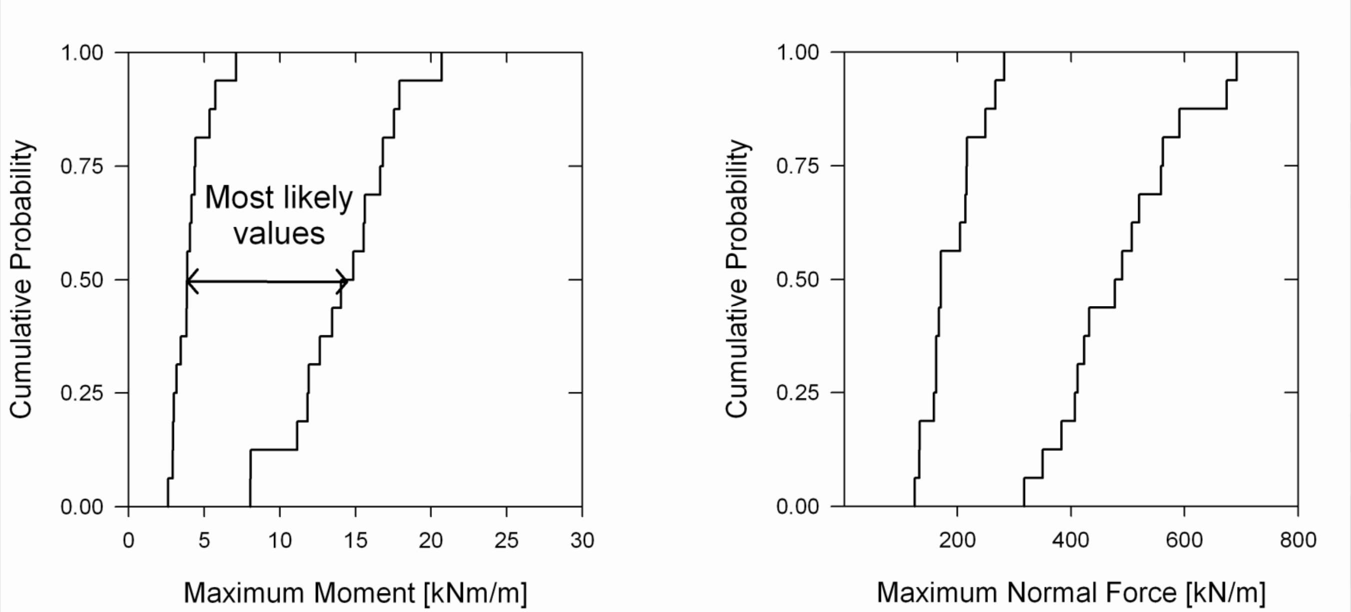
RESULTS – FACTOR OF SAFETY _TH



- Calculated range of factor of safety (FOS) for top heading excavation: approximately between 1.76 and 4.82

- Most likely values between 2.4 and 3.7

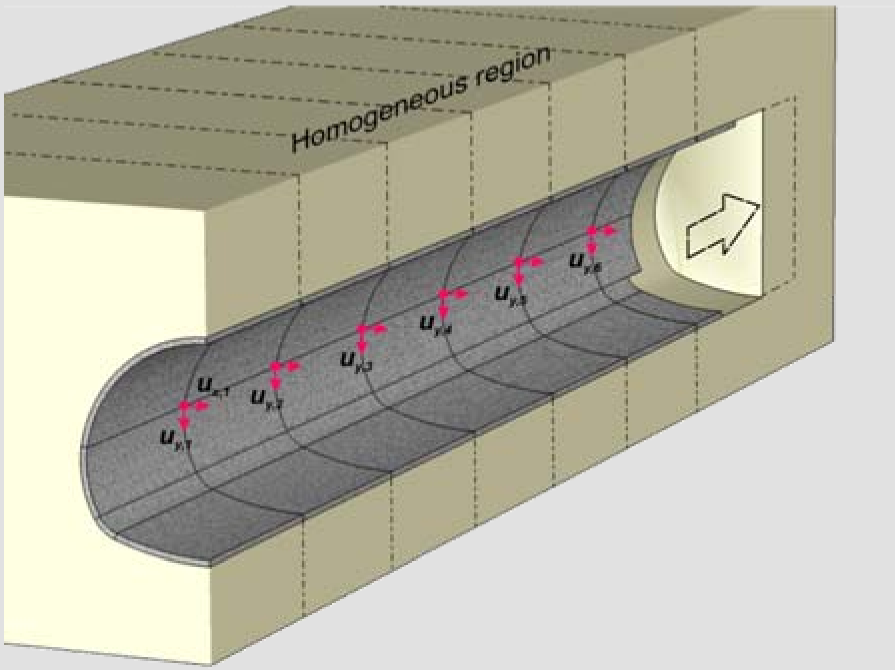
RESULTS – INTERNAL FORCES OF LINING

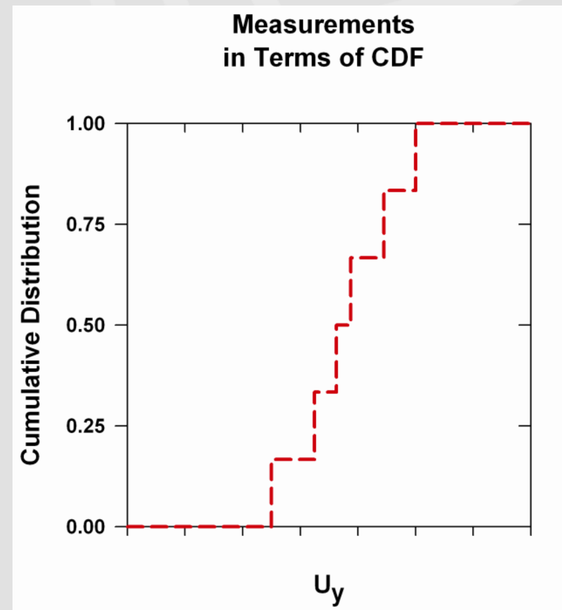


- Range of maximum moment at the final stage: approximately between 2.6 and 20.7
- Most likely values between 3.8 and 14

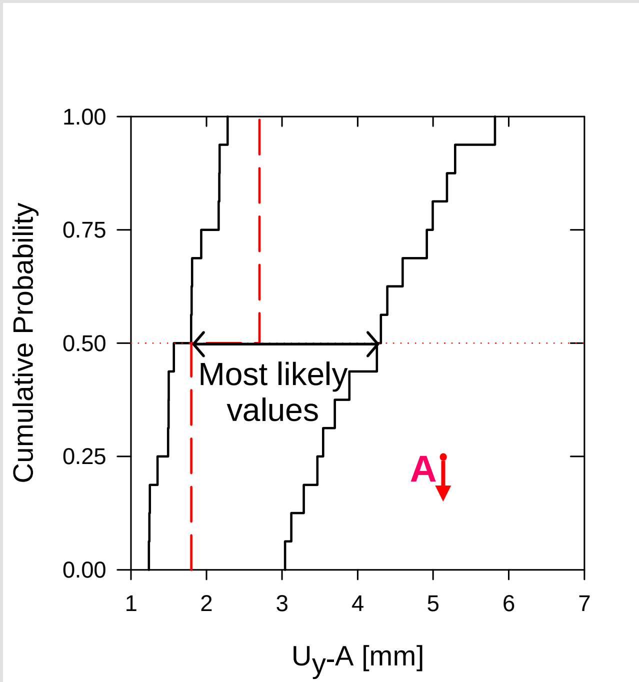
COMPARISON WITH IN-SITU MEASUREMENTS

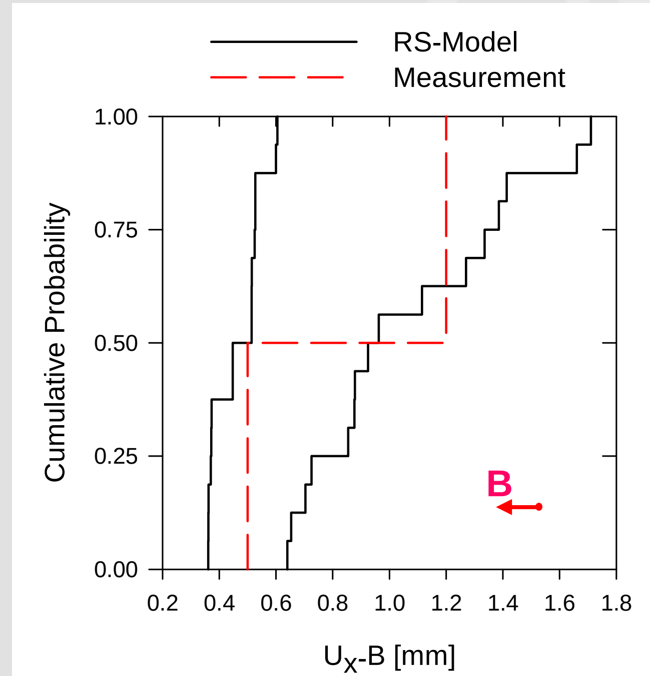
Results of measurements in "homogeneous" section





COMPARISON BETWEEN MEASUREMENTS AND RS-FEM





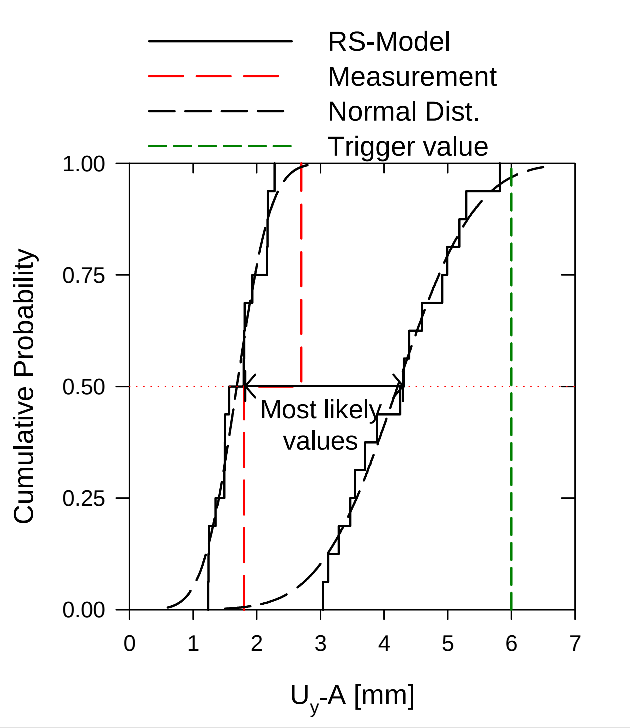
Vertical displacement crown (A)

Horizontal displacement sidewall (B)

- RS-FEM could capture the uncertainty and real response

RELIABILITY ANALYSIS

Vertical displacement crown (A)



Serviceability Limit State Function

$$g(x) = 6 - Uy_A < 0$$

Probability of exceeding a trigger value

	P_f
Max	**0.032**
Min	**> 0.00001**

- No need to assume a distribution type of the responses
- By fitting a distribution to the results, probability measure of crown displacement exceeding a trigger value can be estimated

RELIABILITY ANALYSIS

Assessment of serviceability (SLS) of shotcrete lining according to Schikora & Ostermeier (1988)

$$g(x) = N_{all} - N > 0 \quad \textbf{where} \quad N_{all} = \frac{f_c d}{F_s}\left(1 - 2\frac{e(x) + e_a}{F_s d}\right)$$

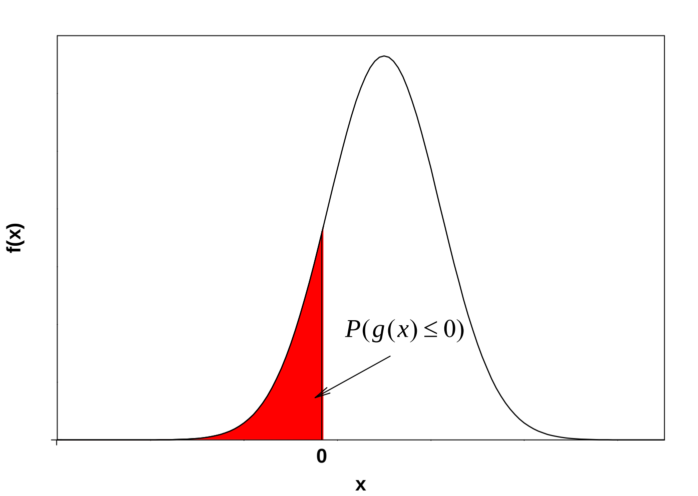
f_c	**Uniaxial strength of shotcrete**
e_a	**Imperfection**
e(x)	**Eccentricity**
d	**Thickness of lining**
M	**Bending Moment**
N	**Axial Force**
F.S.	**Factor of Safety**

$$p_f = p(g(\mathbf{x}) \le 0) = \int_{g(\mathbf{x}) \le 0} f_X(\mathbf{x}) d\mathbf{x}$$

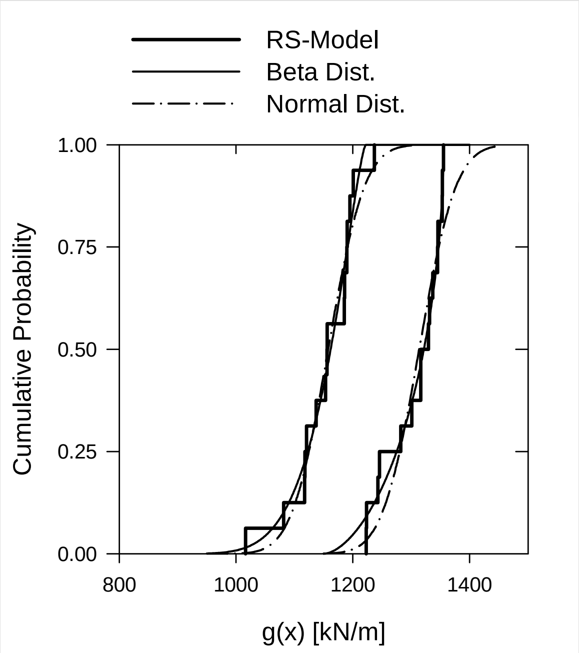
f_X ➔ joint probability density

g ➔ 'limit state function'

$g(x) = r - s = gr(x) - gs(x)$



RELIABILITY ANALYSES



$$g(x) = N_{all} - N > 0$$

	Probability of Failure
Max	**> 0.00001**
Min	**> 0.00001**

SUMMARY

- **RS-FEM provides a user-friendly and practice-oriented framework to predict the system response of a nonlinear structure within a range**
- **Application of RS-FEM needs less computational effort as compared to fully probabilistic methods such as Monte-Carlo simulations**
- **Applicability of the RS-FEM has been demonstrated by a tunnel example in which the field measurements were in reasonable agreement with primary RS-FEM results without any updated data and information during construction**
- **This framework can be considered as a supplementary analysing tool in observational method**

LIMITATIONS

- **If sets are based on very cautious estimates of material parameters range of results can be extremely large and therefore of limited use → everybody involved has to be** - at least to some basic extent - **familiar with the method**
- **If only one set is available for each parameter only an interval is obtained for results but no discrete cumulative density distribution → subsequent reliability analysis not possible**
- **Spatial variability cannot be accounted for in a rigorous manner**

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