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Finite Element Structural Analysis using Imprecise Probabilities Based on P-Box Representation

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Outline

- ***Introduction***
- Interval Monte Carlo Analysis
- Discrete P-box Analysis
- Results
- Conclusions

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Imprimers

- Vladik Kreinovich
- Ray Moore
- A. Neumier
- Scott Ferson

Imprecise probability

- Engineering Design must address optimization of uncertain behavior
- Design requires the prediction of system behavior
- Probability function (CDF, PDF) is needed for analysis of systems with uncertainty

Often, the information needed to accurately define the probability function is not available.

The field of imprecise probability addresses the lack of an accurate CDF.

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Imprecise Probability

- Imprecise Probability (IP) is non an imprecise theory.

"The term 'imprecise probability'—although an unfortunate misnomer .. . enable more accurate quantification of uncertainty than precise probability." (Frank P.A. Coolen, Matthias C.M. Troffaes, Thomas Augustin, International Encyclopedia of Statistical Science, Springer 2010)

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Probability box (Pbox)

- A set of allowable CDF defined by upper and lower bounding functions.

Ferson S, Kreinovich V, Ginzburg L, Myers DS, Sentz K, Constructing probability boxes and Dempster-Shafer structures, Tech. Rep. SAND2002-4015, Sandia National Laboratories, 2003.

- Williamson R. Probabilistic arithmetic. PhD thesis, University of Queensland, Austrialia, 1989.

Pbox representation for Imprecise Probability

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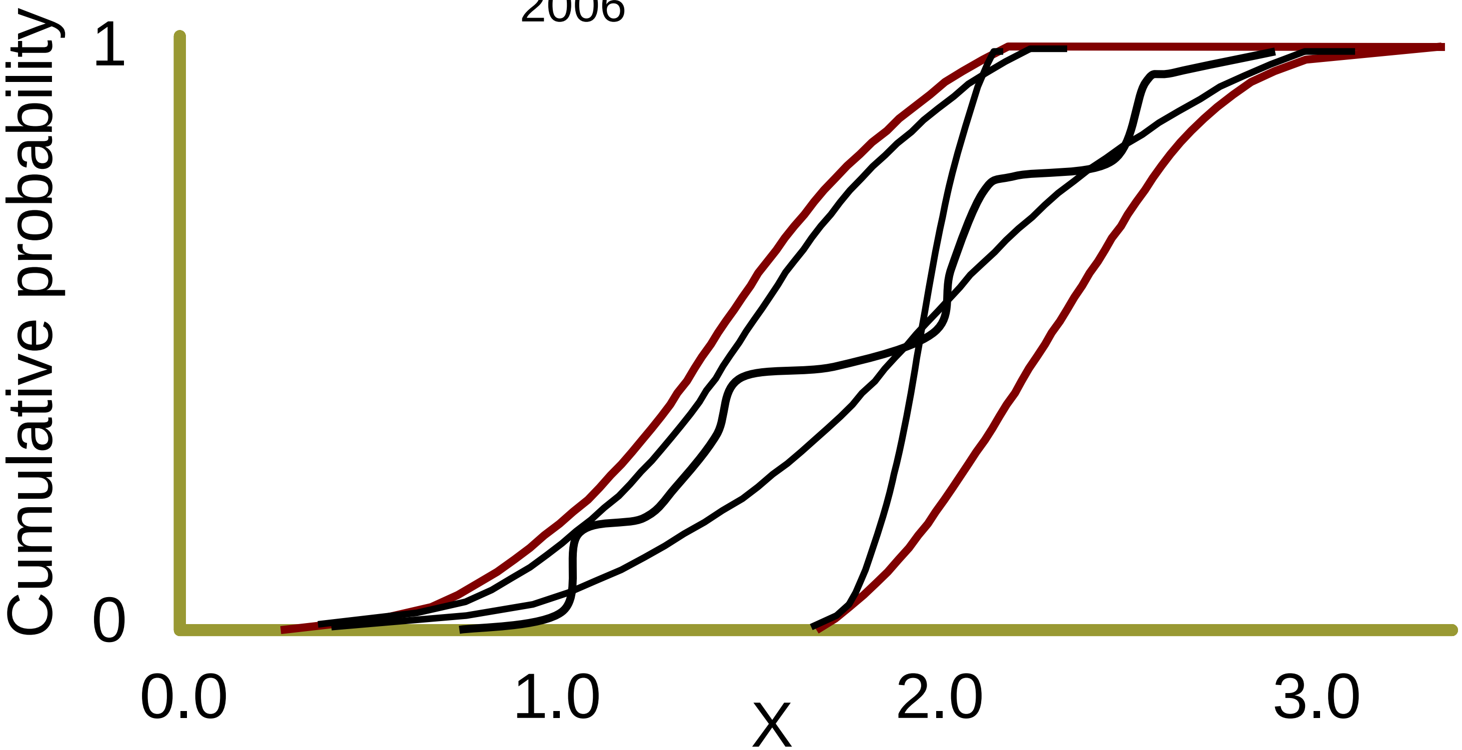
Figure from Scott Ferson and Vladik Kreinovich-REC 2006



Finite Element Analysis

- Approximate methods for solving PDE.
- Reduces continuum problem into a discrete system of equations.
- Interval extension to Finite Element methods have been developed by Muhanna, Hao, Rao, Modares, Berke, Qiu, Elishakoff, Pownuk, Mullen and others.
- We will use a two dimensional truss as an exemplar

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Pbox Monte Carlo method

- Construct interval representation of random input variables from Pbox
- Perform Interval calculation of realization (using Interval Finite Element)
- Collect Interval Response
- Sort Lower and upper bounds and construct Pbox for Response variable.

Assumes independent random inputs.

Zhang, H., Mullen, R. L. and Muhanna, R. L. “Interval Monte Carlo methods for structural reliability”, *Structural Safety*, In press (Online publication complete: 2-FEB-2010,DOI information: 10.1016/j.strusafe.2010.01.001)

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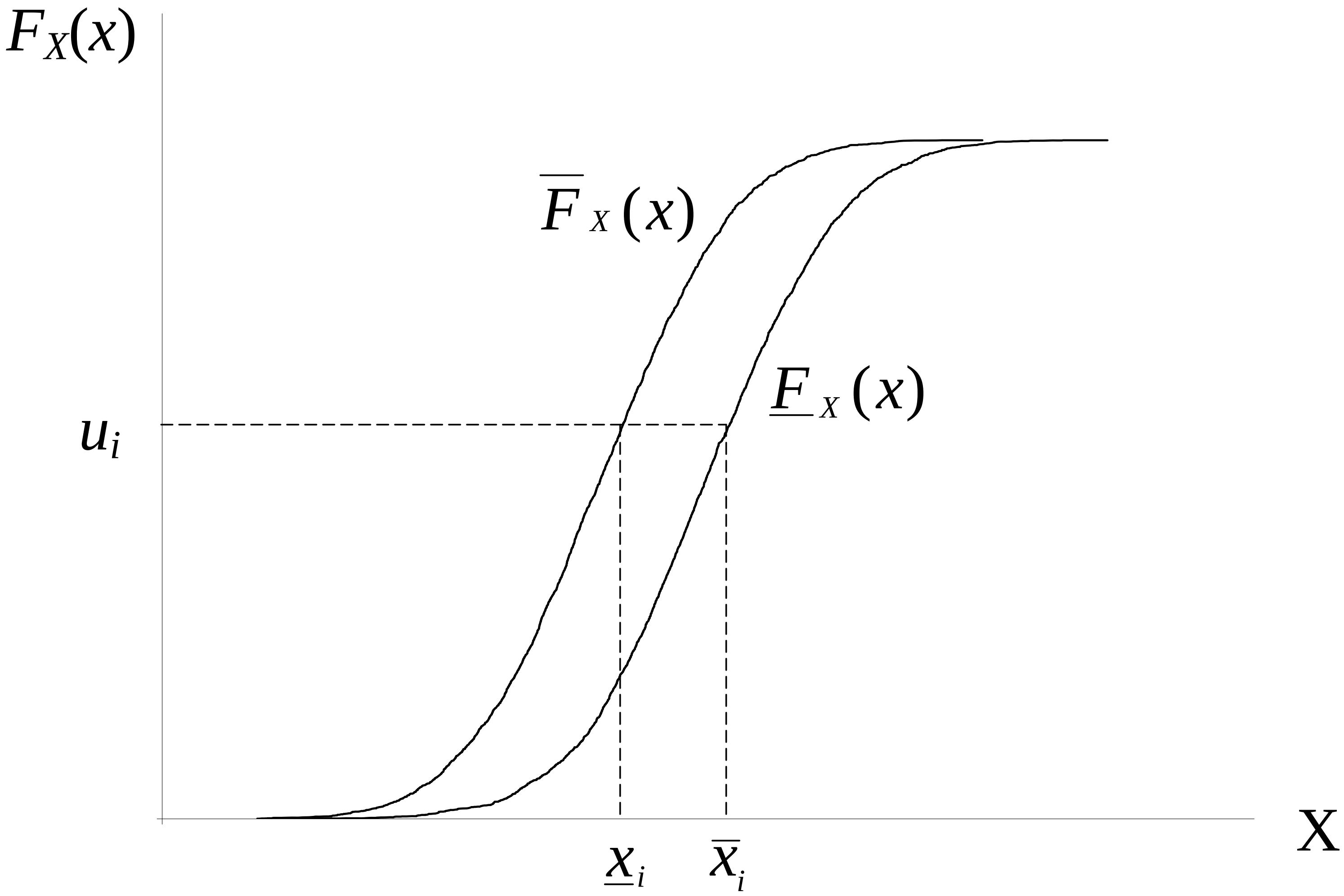
Generation of random intervals from a probability-box



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Discrete Pbox procedure

- Construct Data type for a discrete Pbox
- Define overloaded operators for discrete Pbox variable
- Apply conventional deterministic (scalar) algorithms using Pbox variables
- Result is in the form of a discrete Pbox

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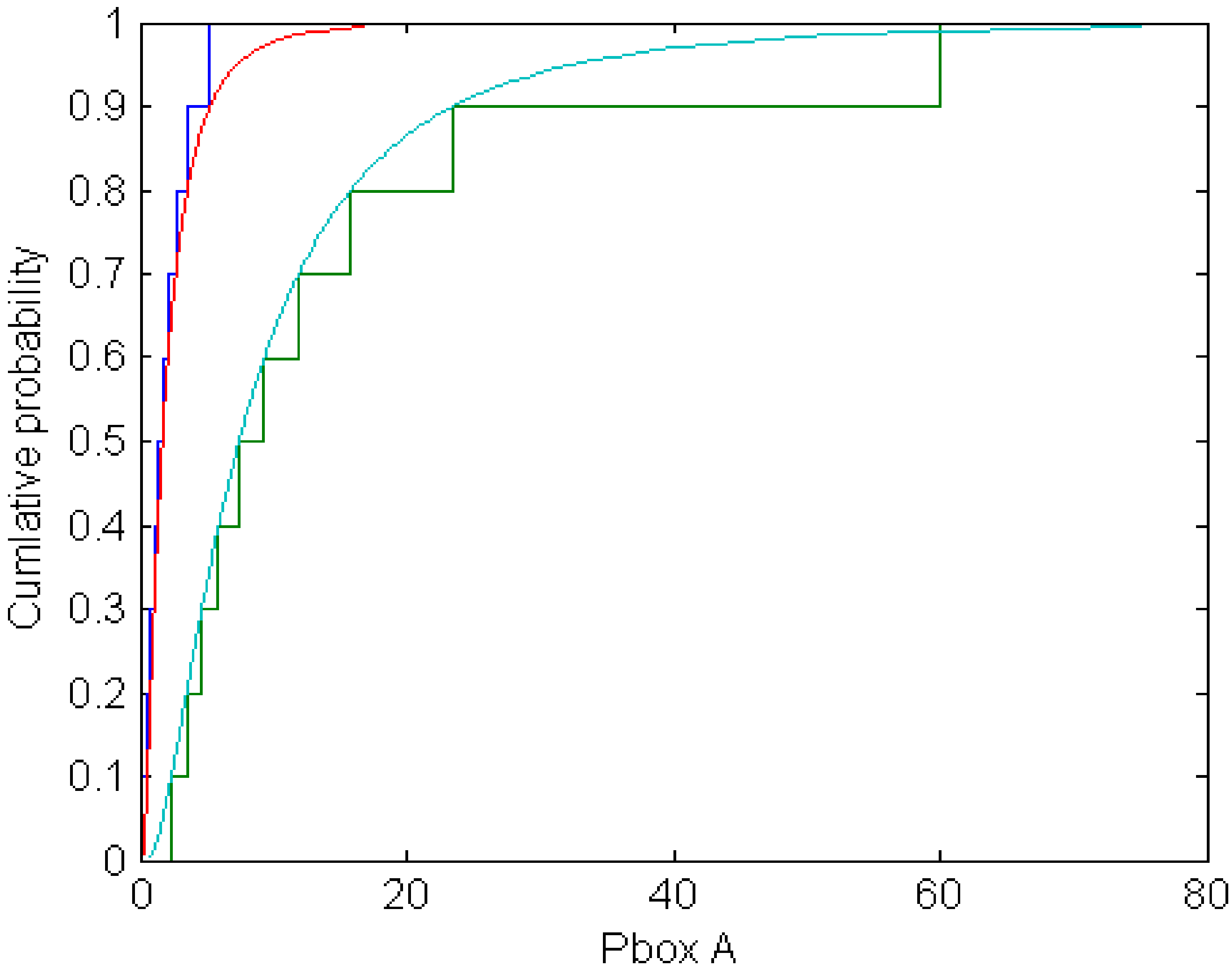
Interval based discrete Pbox

- Proved interval bounds for a given range of the CDF
- Uniform discretization for algorithmic simplicity
- Preserve the guaranteed enclosure philosophy from Interval Arithmetic
- Extensions to higher order discretization and non-uniform discretization are possible.

P-Box defined by bounding lognormal distribution with mean of [2.47, 11.08] and standard deviation of [2.76, 12.38].



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Best fit vs. Guaranteed enclosure

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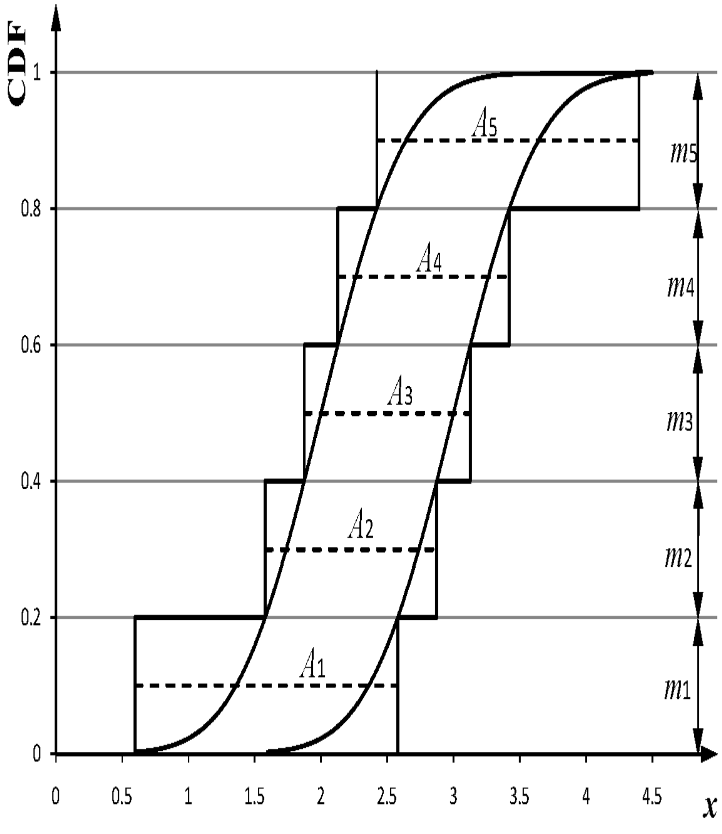
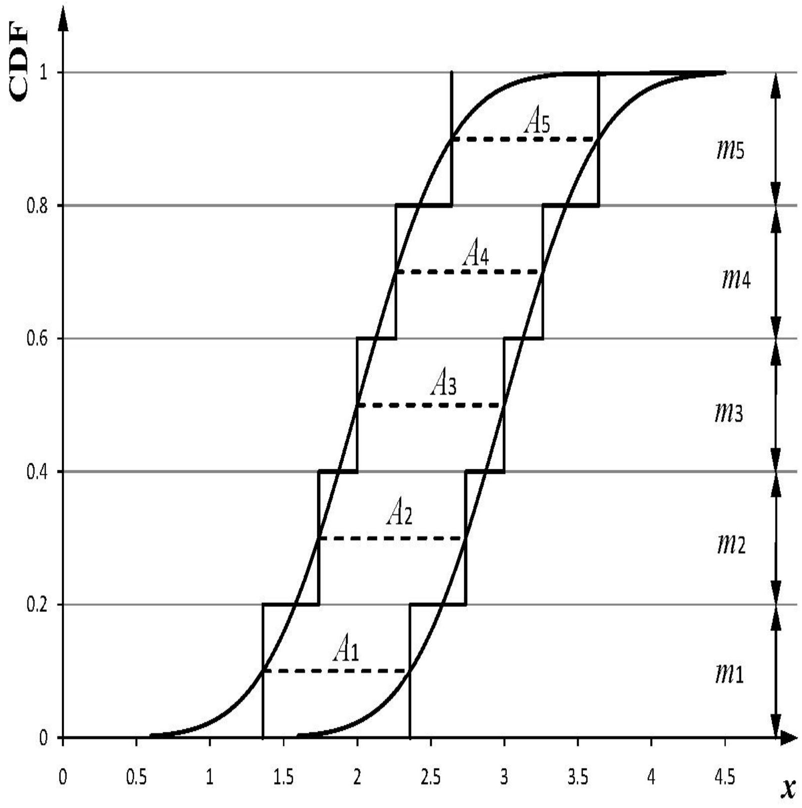
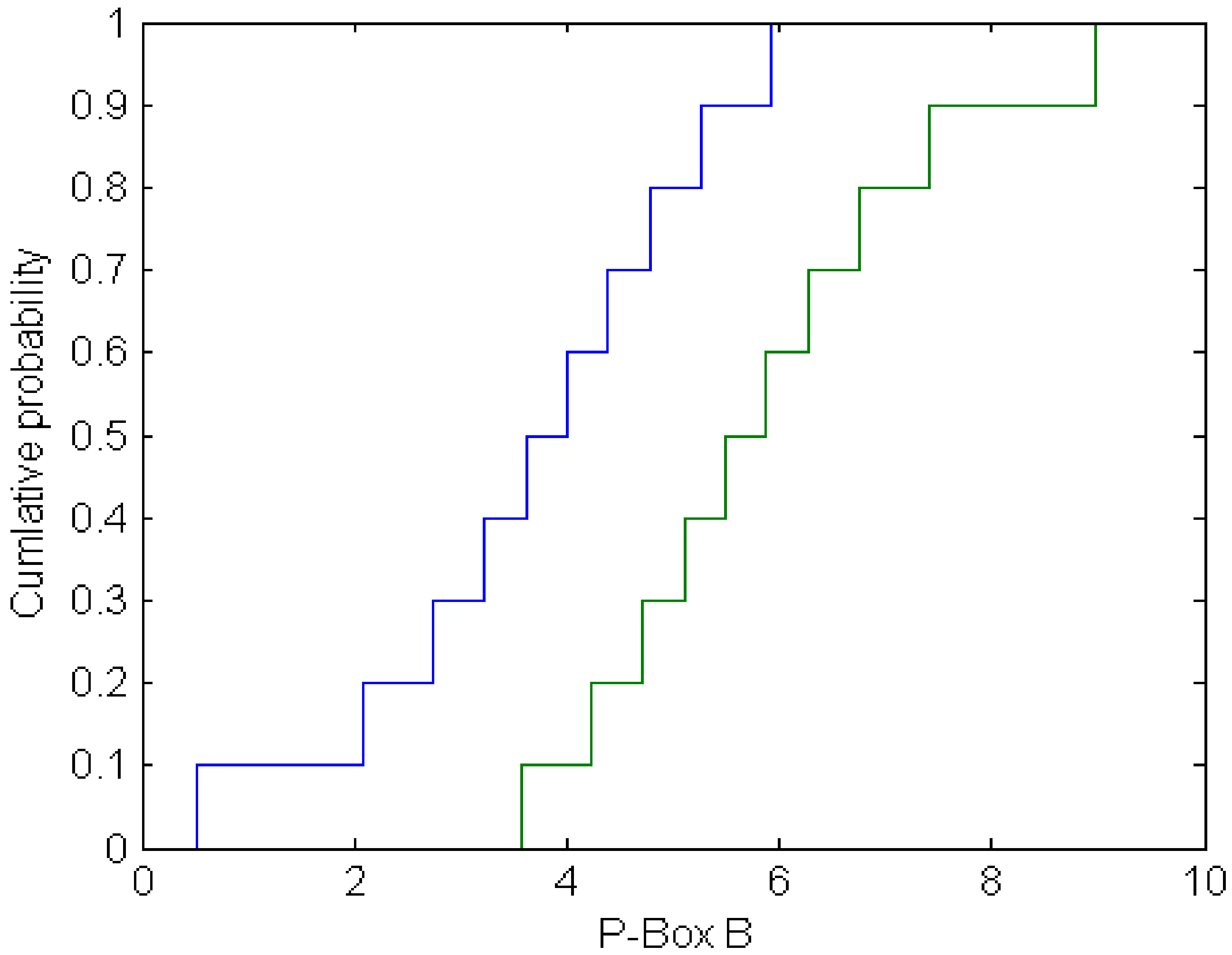




P-Box defined by bounding normal distribution with mean of [4.0, 4.5] and standard deviation of 1.5.



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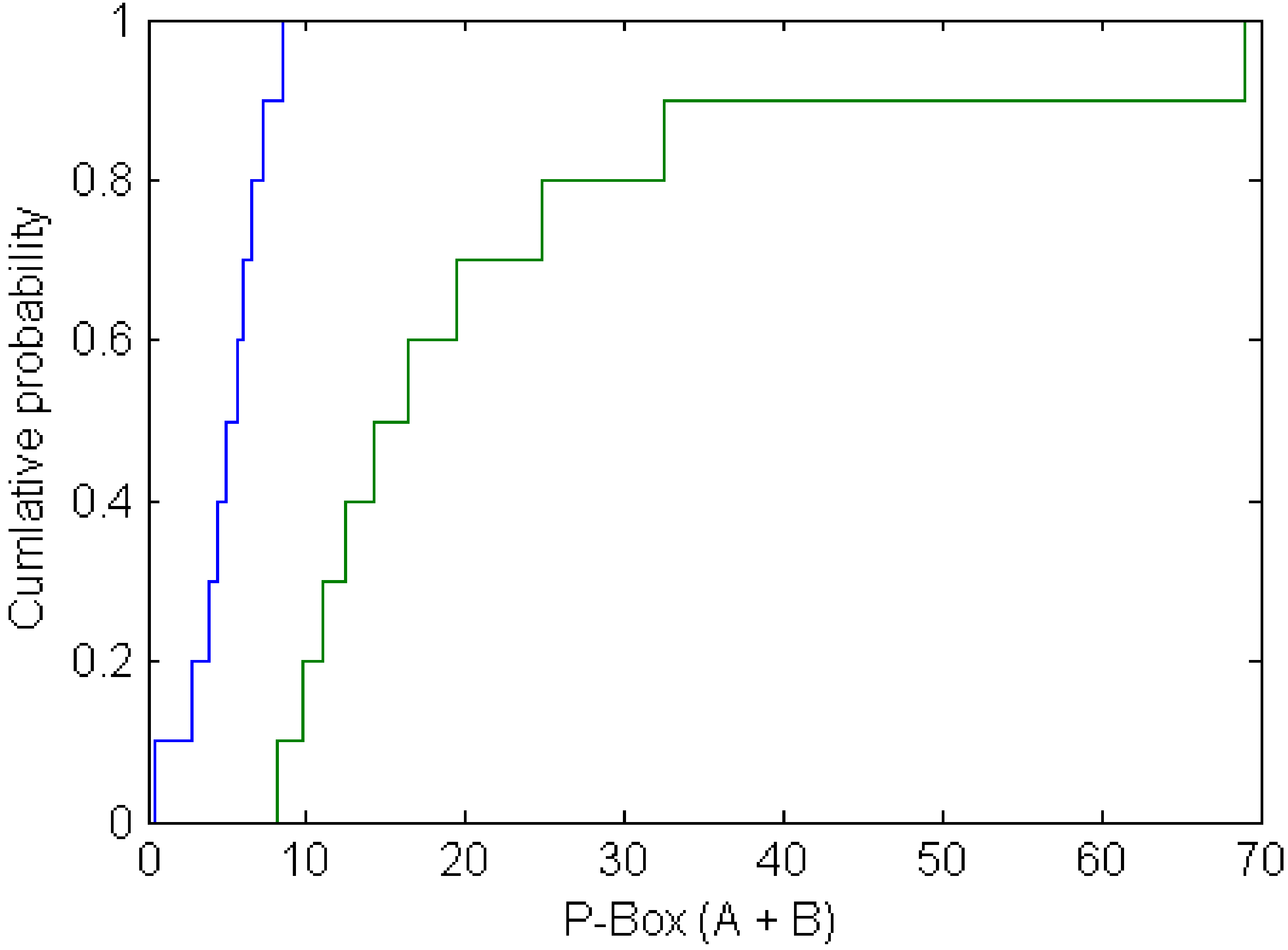
Interval focal elements for P-Boxes A and B and associated probability mass. Cartesian product for A + B

	CDF B	[0,.1]	[.1,.2]	[.2,.3]		[.7,.8]	[.8,.9]	[.9,1.0]
CDF A	A\B	[.51,3.58]	[2.08,4.24]	[2.74,4.72]	…	[4.78,6.77]	[5.26,7.43]	[5.92, 8.99]
[0.,.1]	[0, 2.33]	[.15, 5.91]	[2.08,6.57]	[2.74,7.05]	…	[4.78, 9.1]	[5.26, 9.76]	[5.92, 11.32]
[.1,.2]	[.52, 3.46]	[1.03,7.04]	[2.6,7.7],	[3.26,8.18]	…	[5.3, 10.23]	[5.78, 10.89]	[6.44, 12.45]
[.2,.3]	[.77, 4.61]	[1.28, 8.19]	[2.85,8.85]	[3.51,9.33]	…	[5.55, 1.38]	[6.03, 12.04]	[6.69, 13.6]
	⋮	⋮	⋮	⋮		⋮	⋮	⋮
[.7,.8]	[2.64,15.76]	[3.15,19.34]	[4.72,20.]	[5.3820.48]	…	[7.42, 2.53]	[7.9, 23.19]	[8.56, 24.75]
[.8,.9]	[3.52, 23.4]	[4.03, 26.98]	[5.6,27.64]	[6,26,28.12]	…	[8.3, 30.17]	[8.78, 30.83]	[9.44, 32.39]
[.9,1.0]	[5.22, 60.]	[5.73, 63.58]	[7.3,64.24	[7.96,64.72]	…	[10., 66.77]	[10.48, 67.4]	[11.44, 68.9]

A+B



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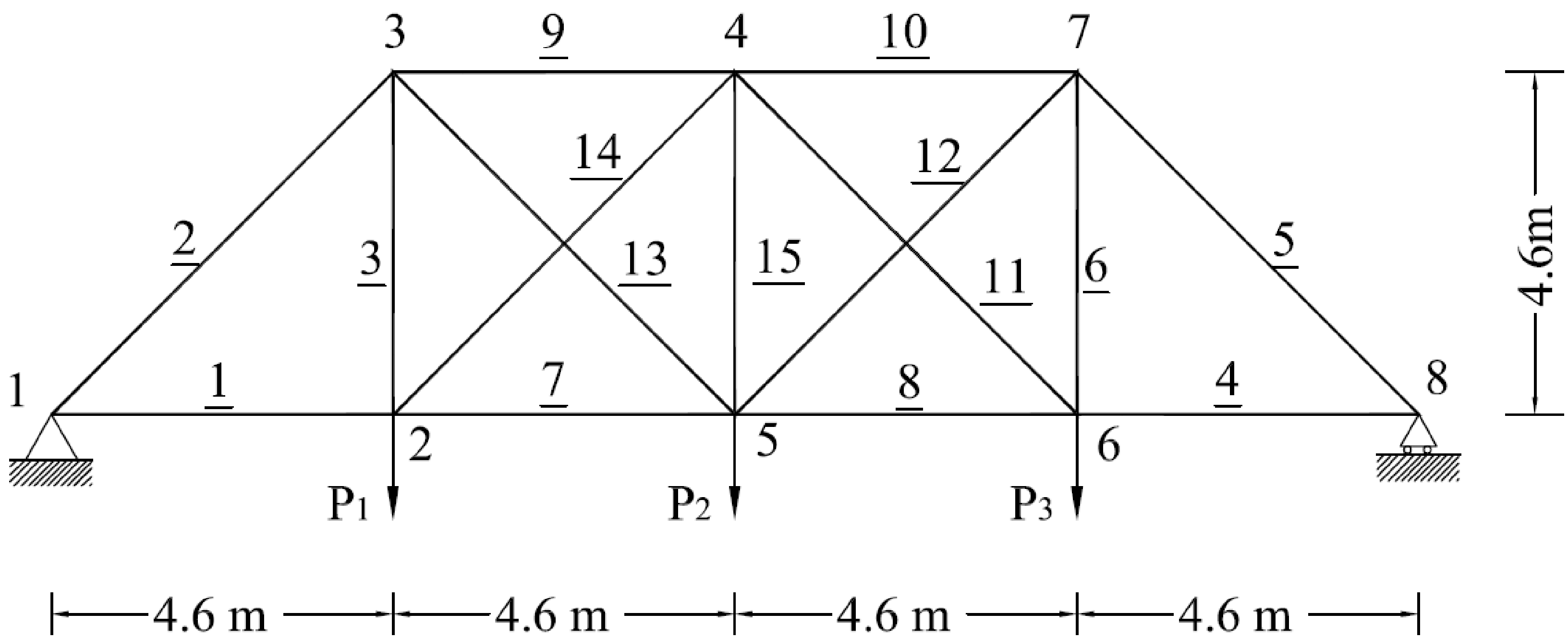
A truss structure



Property	Value
cross-sectional areas for elements A_1 to A_6	10.32 cm^2
cross-sectional areas for elements A_7 to A_{15}	6.45 cm^2
Elastic modulus	200 GPa

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Statistics of random loadings acting on the truss		
	90% confidence interval	99%confidence interval
Mean ln P1	[4.4465, 4.5199]	[4.4258, 4.5407]
Mean ln P2	[5.5452, 5.6186]	[5.5244, 5.6393]
Mean ln P3	[4.4465, 4.5199]	[4.4258, 4.5407]
ln Standard dev. P1, P2, P3	0.09975	0.09975

Central deflection of the truss lower cord -.Interval Monte Carlo results for 90% confidence interval			
Number MC simulations	Mean of the deflection of the center of the lower cord	Variance * 10^5 of the central deflection	Probability of failure
5,000	[-0.0619094, -.057528]	[1.7665, 2.0459]	[0.0004, .0042]
50,000	[-0.0619236, -.0575412]	[1.7833, 2.06539]	[.00024, .00446]
500,000	[-.0619299, -.057547]	[1.7724, 2.053]	[.000172, .004648]

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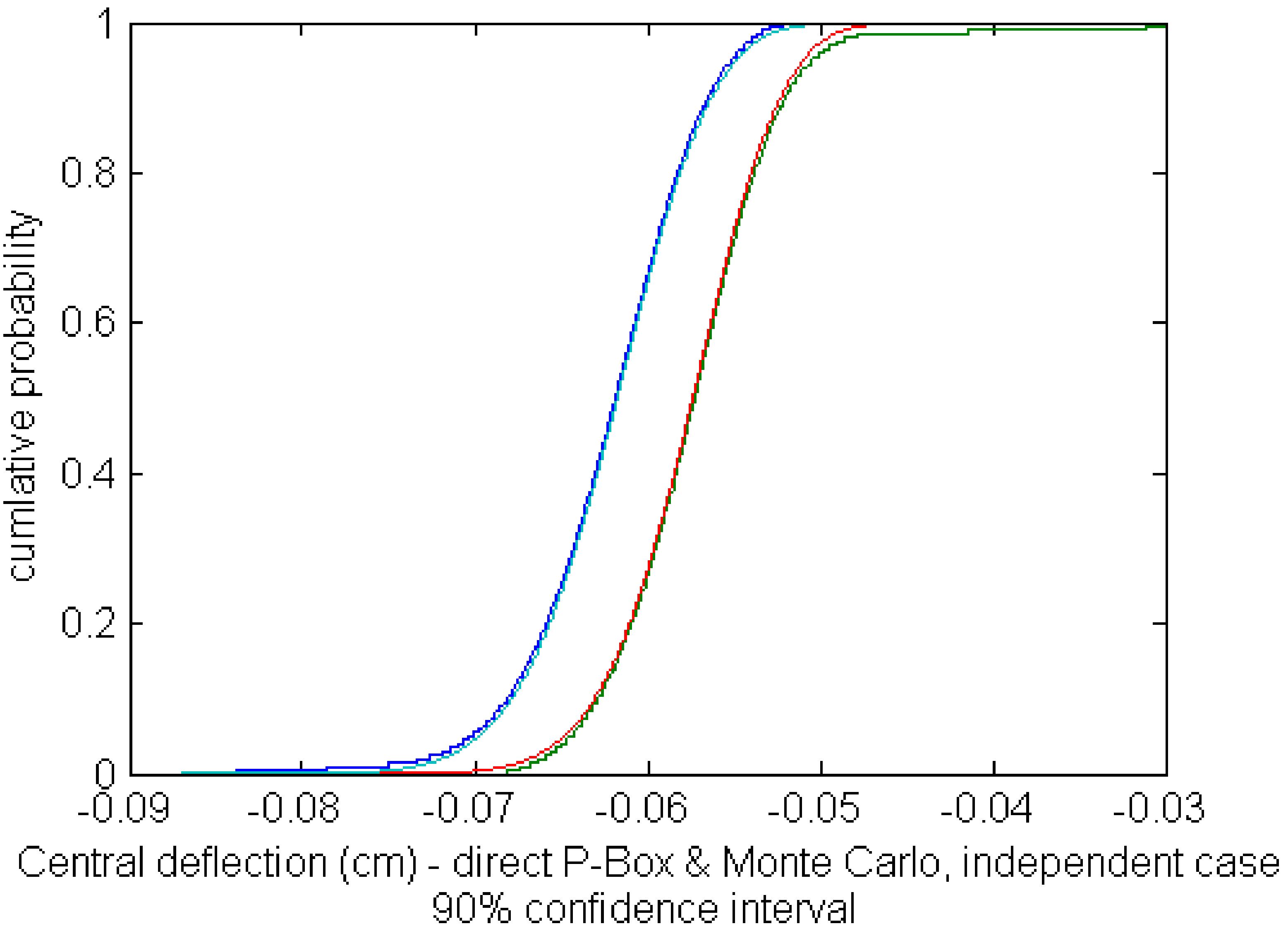

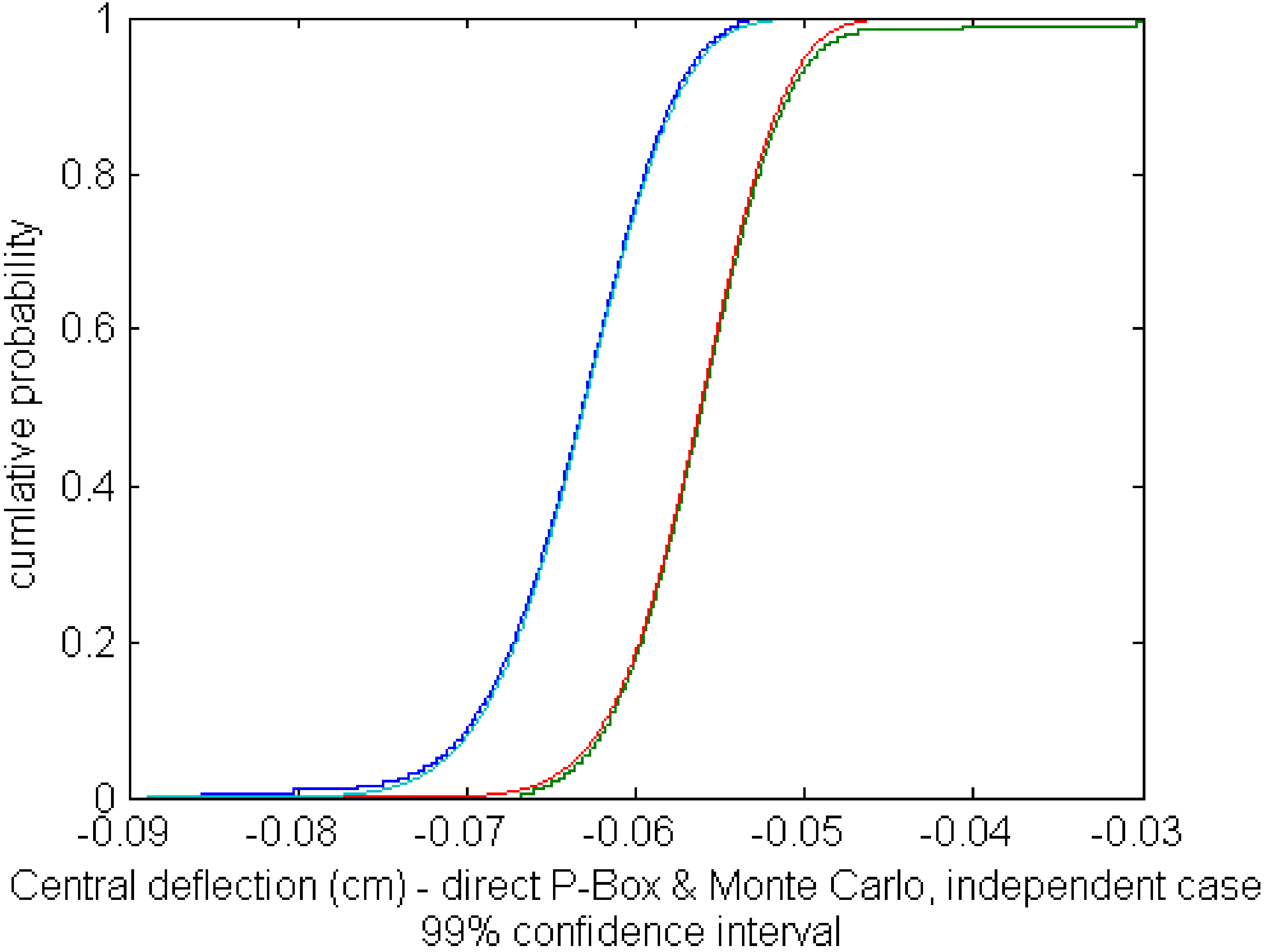

1
0.8
0.6
0.4
0.2
0
cumlative probability
-0.09
-0.08
-0.07
-0.06
-0.05
-0.04
-0.03
Central deflection (cm) - direct P-Box & Monte Carlo, independent case
99% confidence interval

Table 4. Bounds on the mean of the central deflection for different P-Box discretization levels. 90% confidence interval.

Number of P-Boxs in Discretization	Mean independent	Mean any dependency	Mean Moment Equations
200	[-0.062197, -0.056952]	[-0.066677, -0.05329]	[-0.061625, -0.057264]
100	[-0.062432, -0.056385]	[-0.066699, -0.05291]	[-0.061625, -0.057264]
50	[-0.062971, -0.055293]	[-0.066993, -0.052136]	[-0.061625, -0.057264]
25	[-0.063893, -0.053213]	[-0.067388, -0.050566]	[-0.061625, -0.057264]
MC	[-0.0619123, -.0575307] (50,000 realization)		

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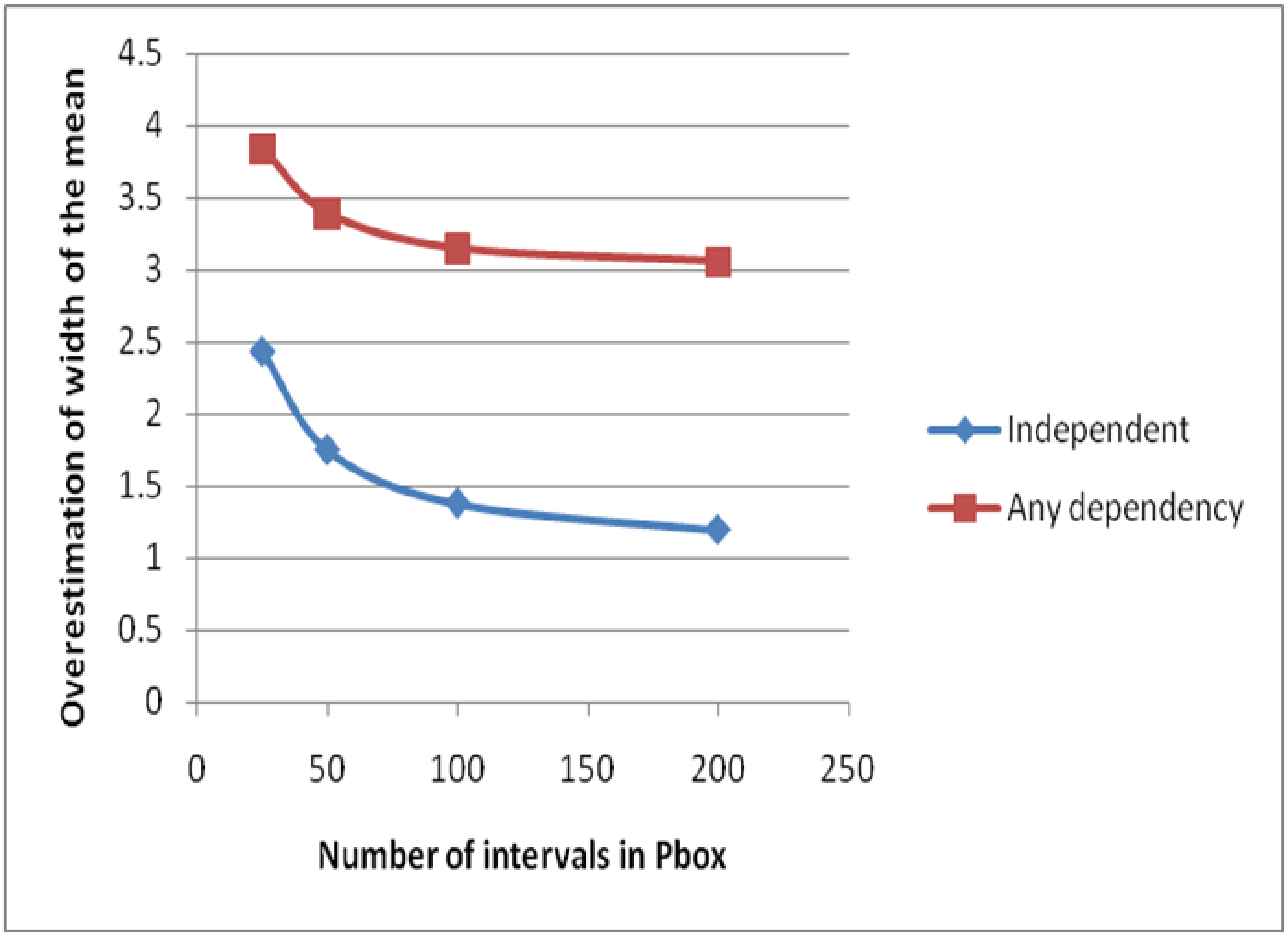

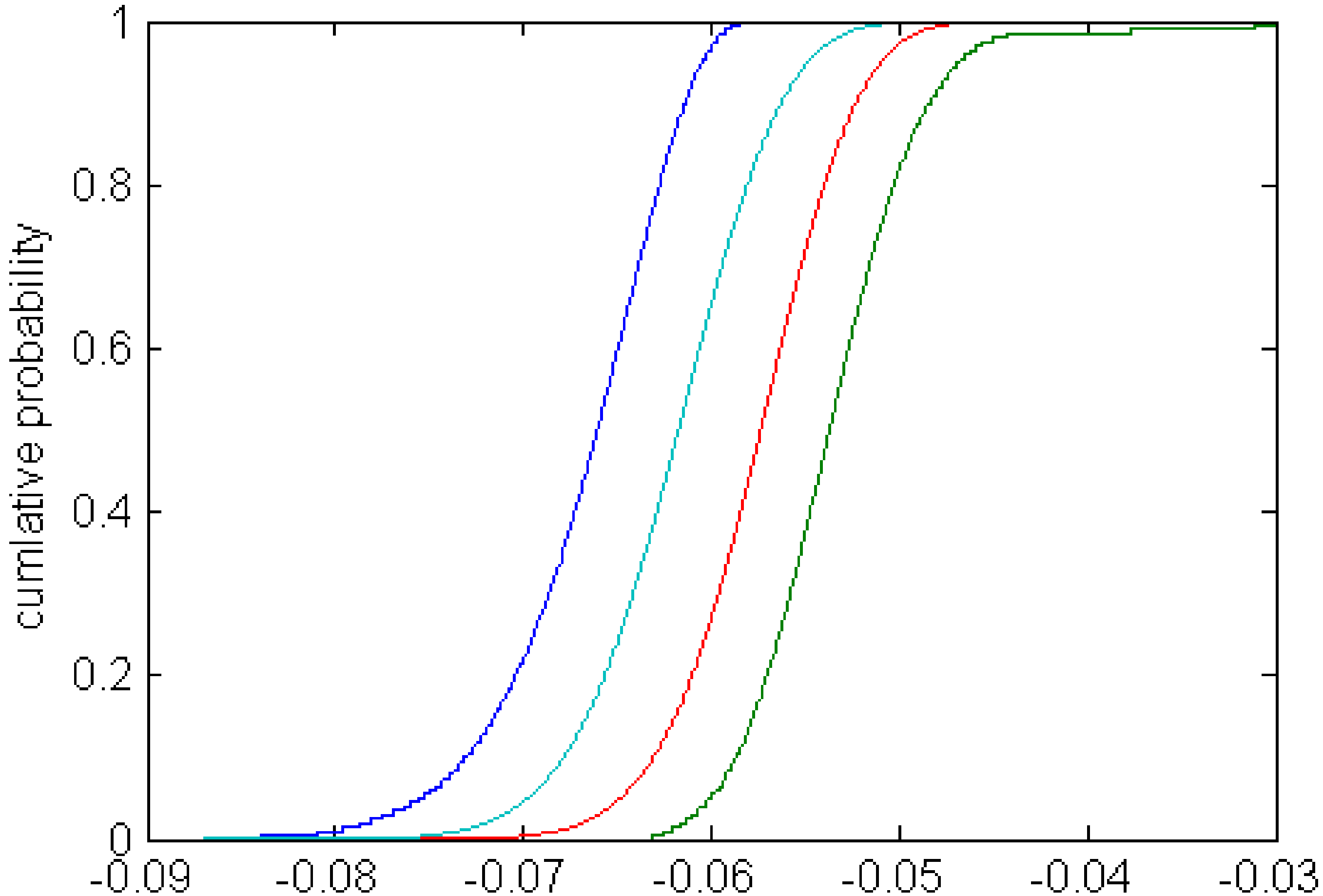

Central deflection (cm) - direct P-Box method for independent and any dependency cases- 90% confidence inteval

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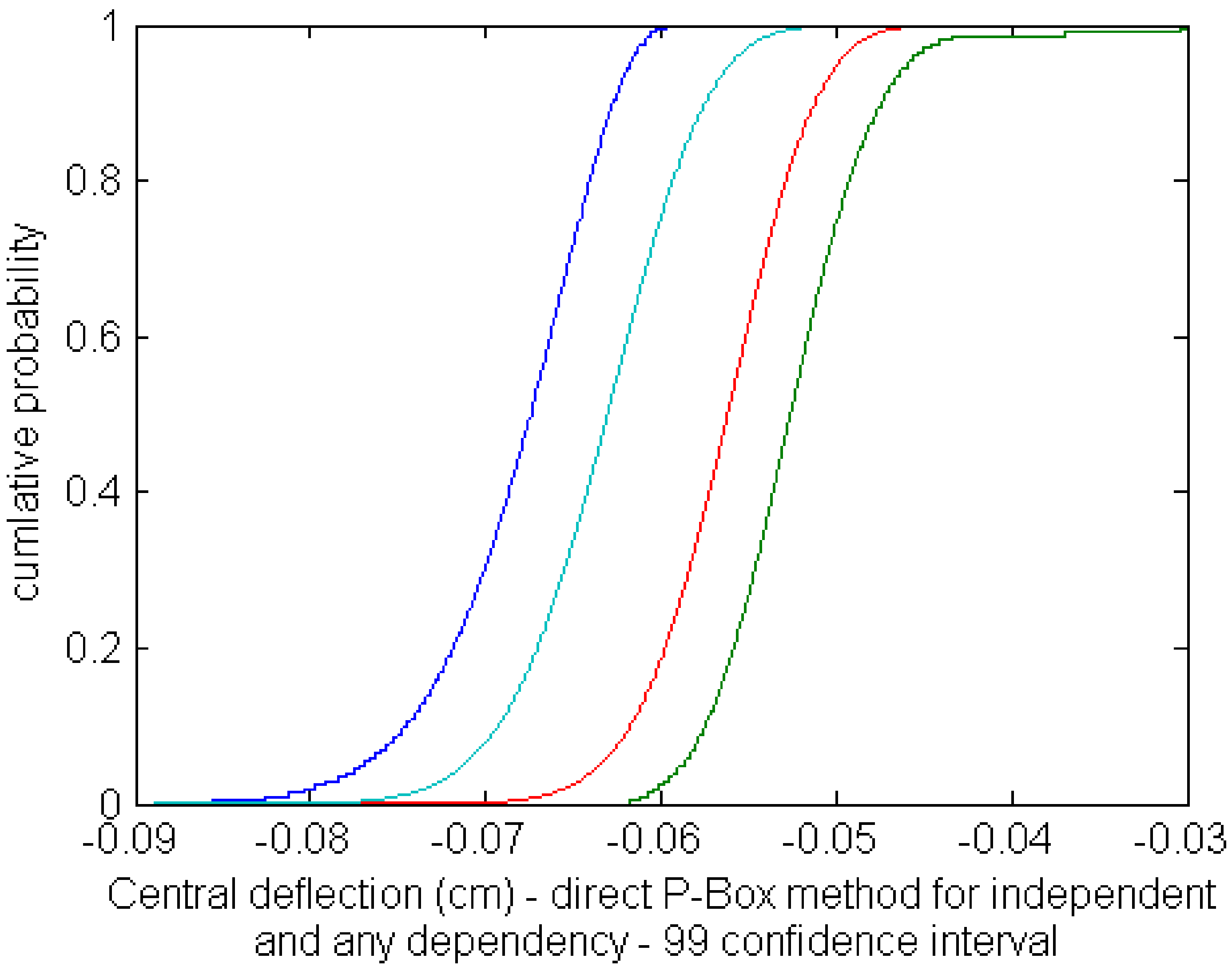

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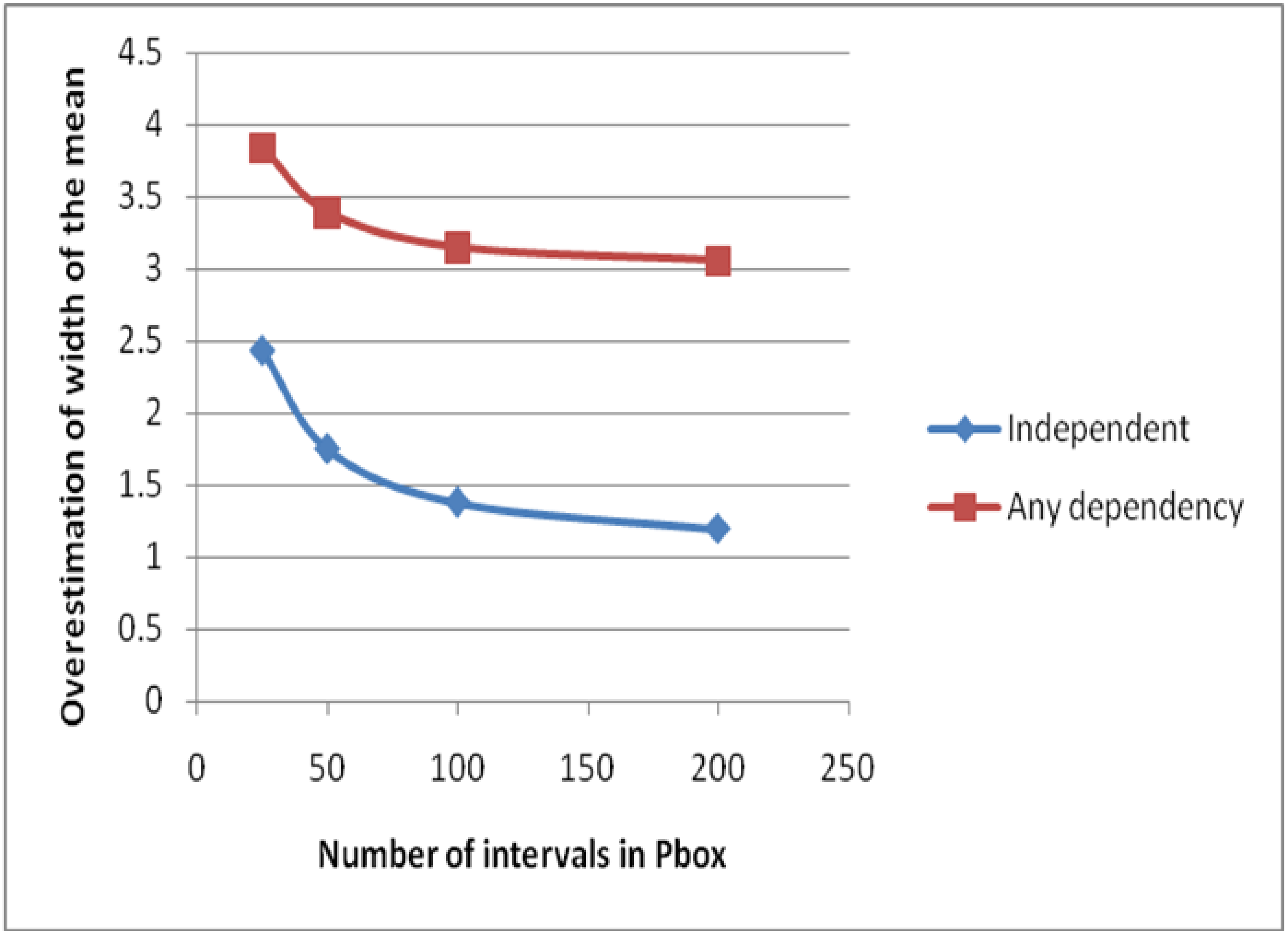

Computational time for Monte Carlo and discrete P-Boxes	
Method	CPU time, seconds.
Monte Carlo 50,000 realizations	17
Monte Carlo 500,000	180
Discrete P-Box 100 intervals independent	13
Discrete P-Box 100 intervals any-dependency	1
Discrete P-Box 200 intervals independent	99
Discrete P-Box 200 intervals any-dependency	3

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Conclusions

- A solution using discrete P-Box structures for uncertain loading in FEM analysis is developed.
- The discrete P-Box method is compared with an interval Monte Carlo based P-Box analysis under the assumption of independent random variables.
- In the problem considered, the discrete P-Box and Monte Carlo produced very similar results for the structural response.
- The computational efforts were also comparable.

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Conclusions

- However, discrete P-Box methods can also examine behavior with other dependencies. In particular, results are also presented for the case of **any dependency** between random variables.
- In the truss considered, the bounds on the CDF of the structural response are wider under the any-dependency assumption compared to assuming independence.
- The computational effort required for the any-dependency assumption is significantly less than both the Monte Carlo and discrete P-Box methods when independent variables are assumed.

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QUESTIONS?