Reliability-based design by adaptive quantile estimation

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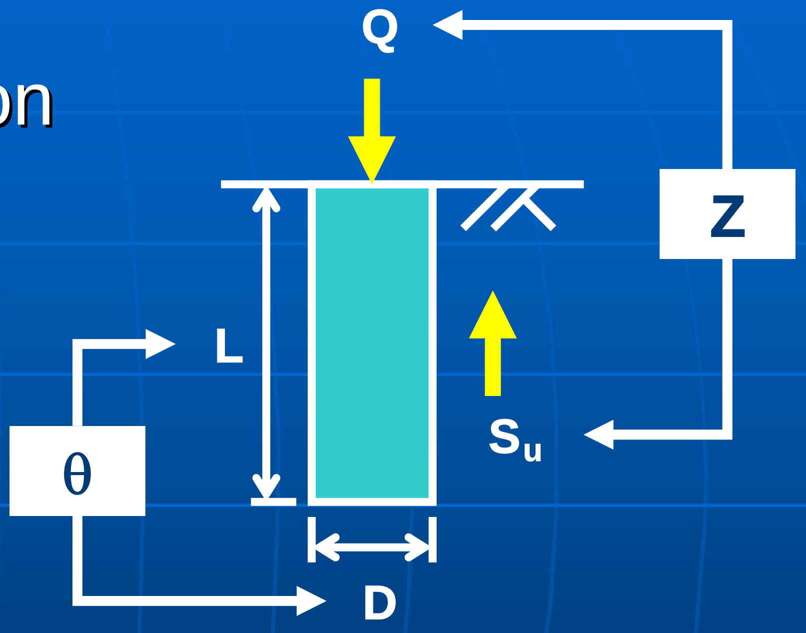
Outline

- Quantile vs RBD
- Estimating quantile as a function of design parameters
- Adaptive quantile estimation
- Examples

Quantile vs RBD

- $R(Z,\theta)$: performance function
  - Z: random variables
  - θ: design parameters
  - R>1: failure



- Example – pile
  - Z: uncertain soil properties
  - θ: length and diameter of the pile
  - $R(Z,\theta) = Load(Z,\theta)/Capacity(Z,\theta)$
  - $R_n(\theta) = Load(Z_n,\theta)/Capacity(Z_n,\theta)$
  - $G(Z,\theta) = R(Z,\theta)/R_n(\theta)$

Quantile vs RBD

- IF $\eta^*(\theta)\cdot R_n(\theta)\le 1 \Longleftrightarrow P\left(R[Z,\theta]>1\middle|\theta\right)\le P_F^*$

  equivalent

- Then $P\left(\dfrac{R[Z,\theta]}{R_n(\theta)}\le \eta^*(\theta)\right)=P\left(G[Z,\theta]\le \eta^*(\theta)\right)=1-P_F^*$

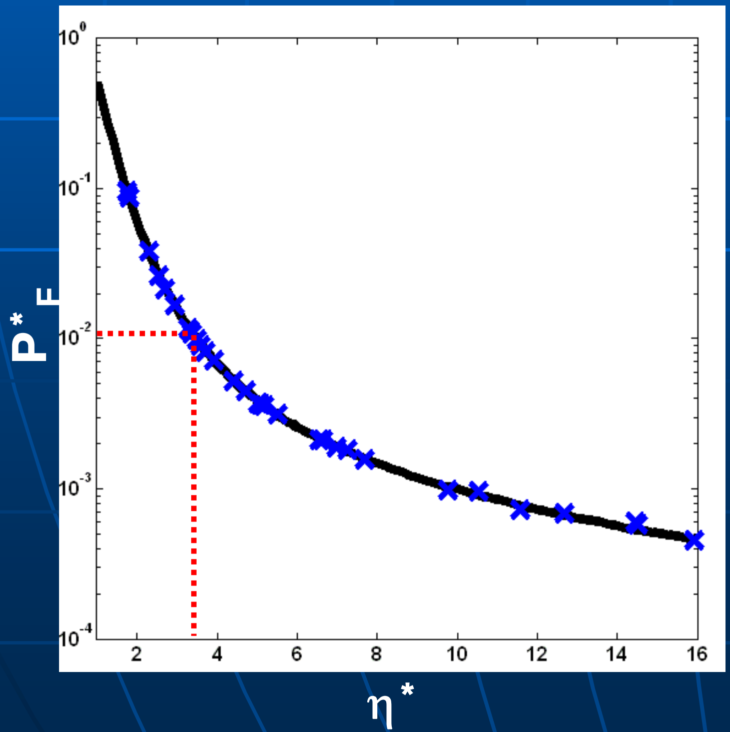
- That is, $\eta^*(\theta)$ is the $1-P_F^*$ quantile of $G[Z,\theta]$

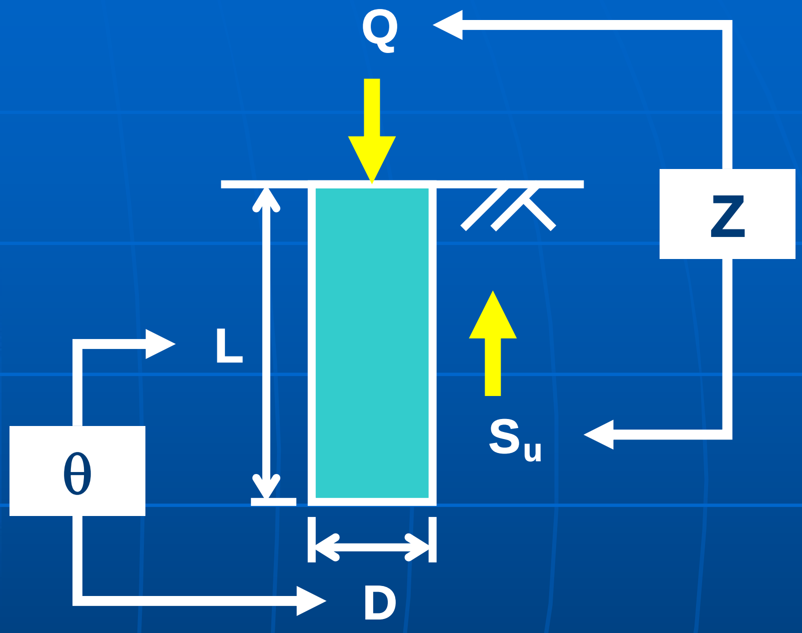
Quantile vs RBD

- In other words, if the $1-P_F^*$ quantile of $G[Z,\theta]$ can be found as $\eta^*(\theta)$, the reliability design constraint $P\left(R[Z,\theta]>1\middle|\theta\right)\le P_F^*$ can be transformed into $\eta^*(\theta)\cdot R_n(\theta)\le 1$

Quantile vs RBD

- Example – pile
  - $R(Z,\theta) = L(Z,\theta)/C(Z,\theta)$
  - $R_n(\theta) = L(Z_n,\theta)/C(Z_n,\theta)$





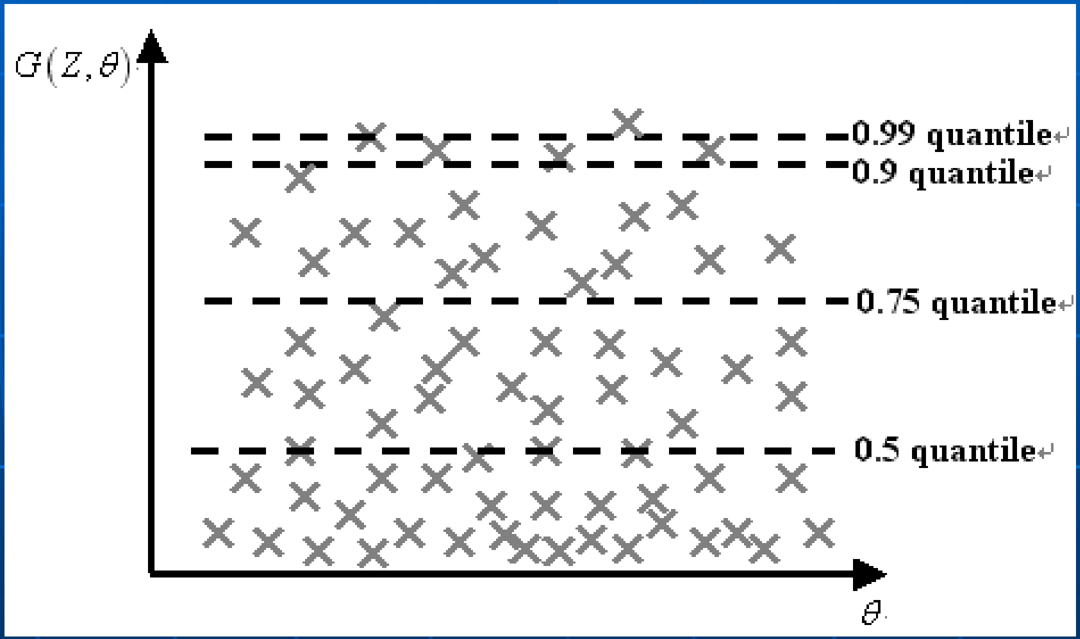
$$P\left(R[Z,\theta] > 1 \middle| \theta\right) \leq 0.01$$

⇕ equivalent

$$R_n(\theta) \leq \frac{1}{3.4}$$

Simple cases

- $\eta^*(\theta)$ is a constant

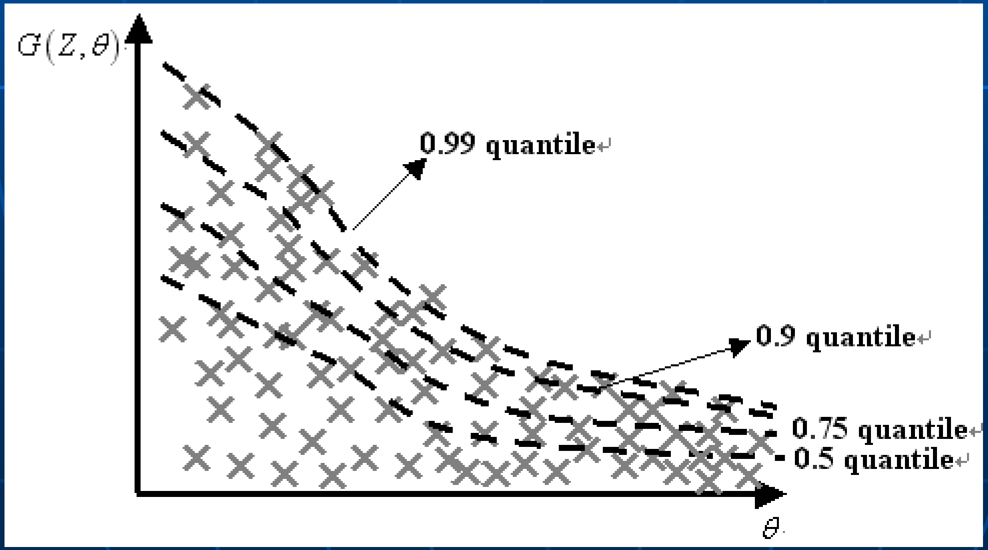


- Estimation of $\eta^*(\theta)$ is simple → MCS can do

Hard cases

- $\eta^*(\theta)$ may not be a constant

- How do we estimate $\eta^*(\theta)$ as a function of θ?

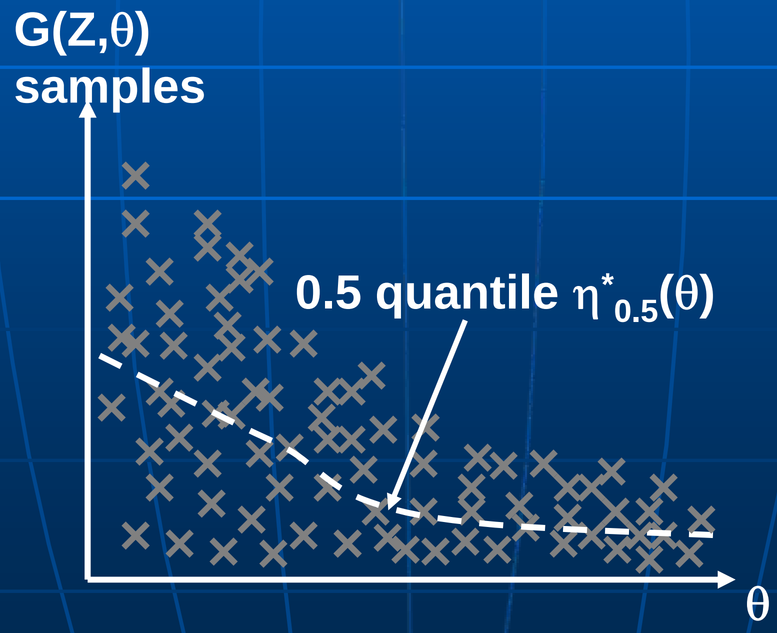


Adaptive quantile estimation

- Use subset simulation to adaptively generate tail samples
- Use generalized Pareto distribution (GPD) to adaptively estimate the tail based on the tail samples
  - GPD is excellent in matching tails

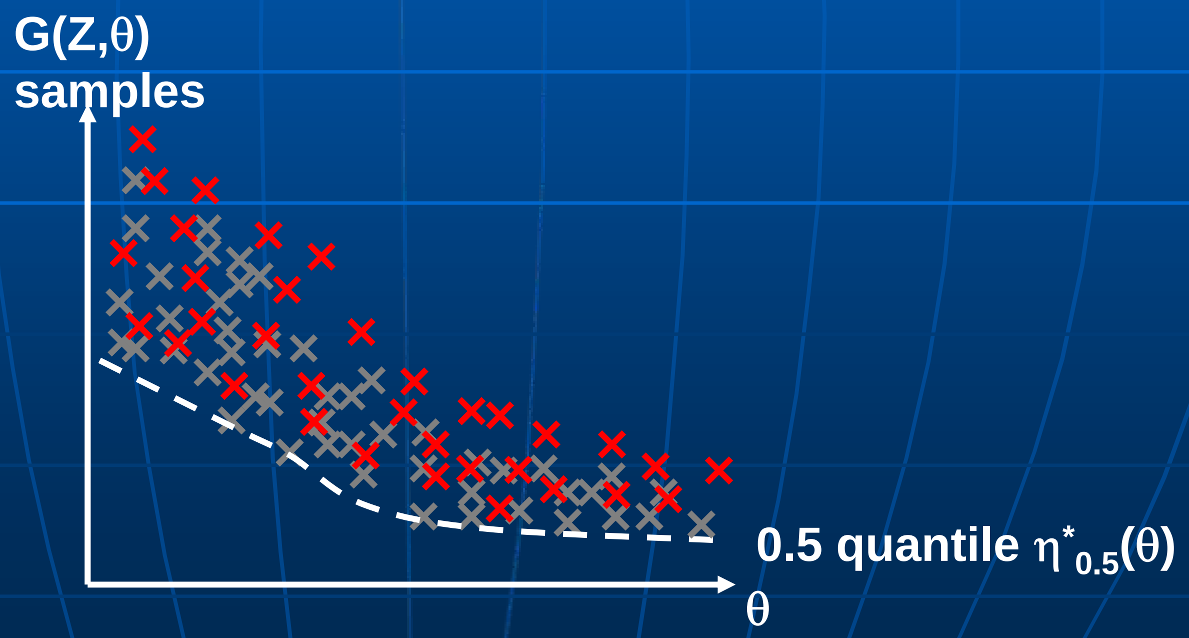
Adaptive quantile estimation

- Step 1:
  - Use generalized Pareto distribution to find $\eta^*_{0.5}(\theta)$



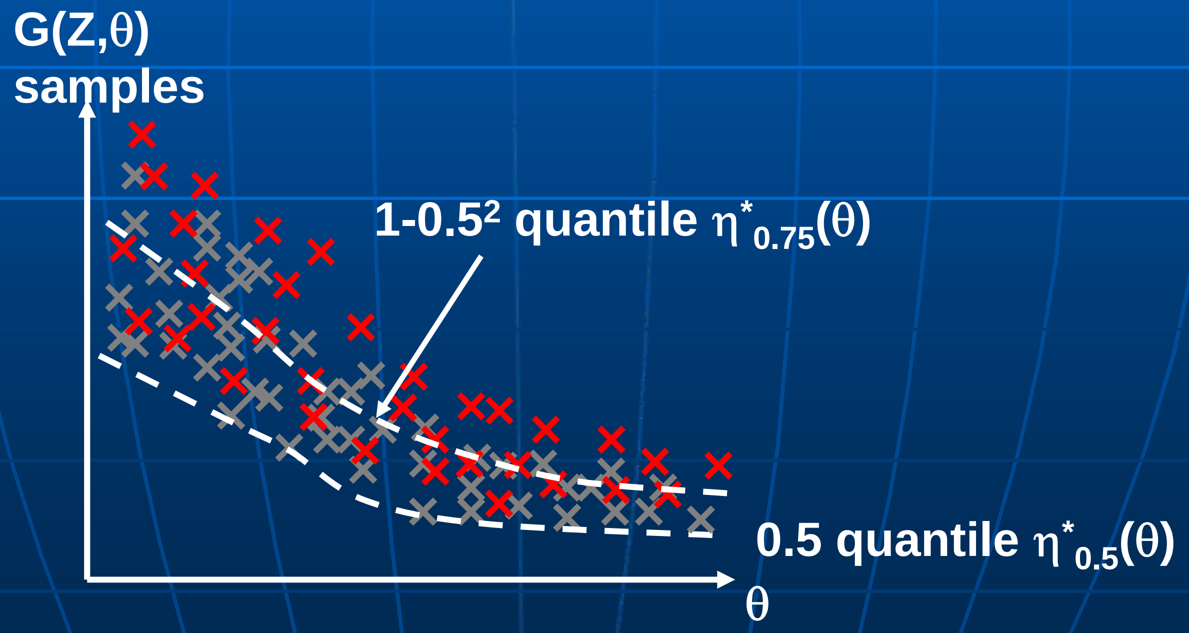
Adaptive quantile estimation

- Step 2:
  - Use subset simulation (Metropolis) to populate the tail region above $\eta^*_{0.5}(\theta)$



Adaptive quantile estimation

- Step 3:
  - Use generalized Pareto distribution to find $\eta^*_{0.75}(\theta)$



Adaptive quantile estimation

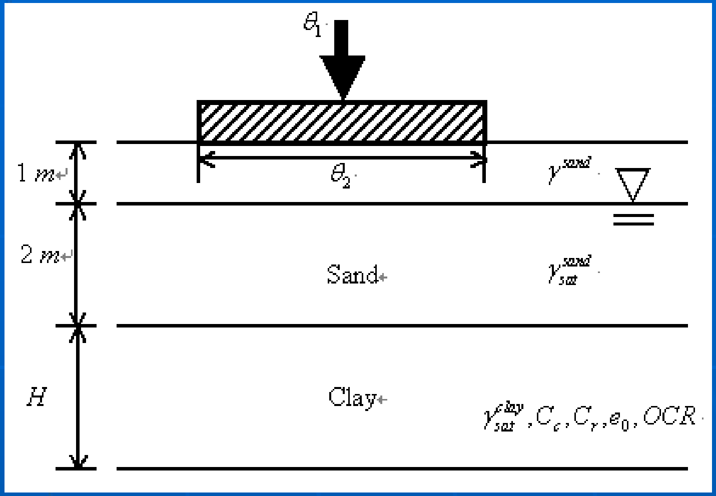
- Continue doing so will get
  - $(P^*_F=0.5) \rightarrow \eta^*_{0.5}(\theta)$
  - $(P^*_F=0.25) \rightarrow \eta^*_{0.75}(\theta)$
  - $(P^*_F=0.125) \rightarrow \eta^*_{0.875}(\theta)$
  - $(P^*_F=0.0625) \rightarrow \eta^*_{0.9375}(\theta)$
  - $(P^*_F=0.03125) \rightarrow \eta^*_{0.96875}(\theta)$
  - …
  - $(P^*_F=0.5^m) \rightarrow \eta^*_{1-0.5^m}(\theta)$

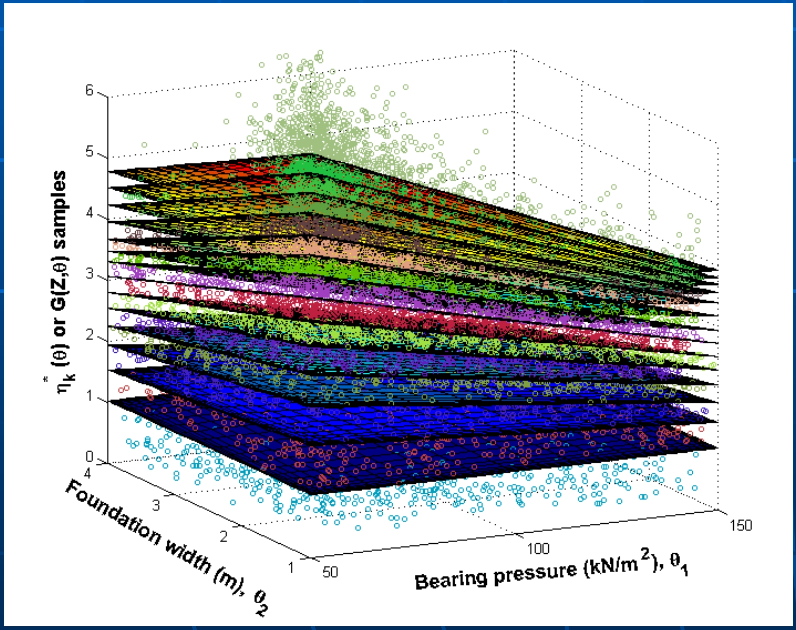
Adaptive quantile estimation

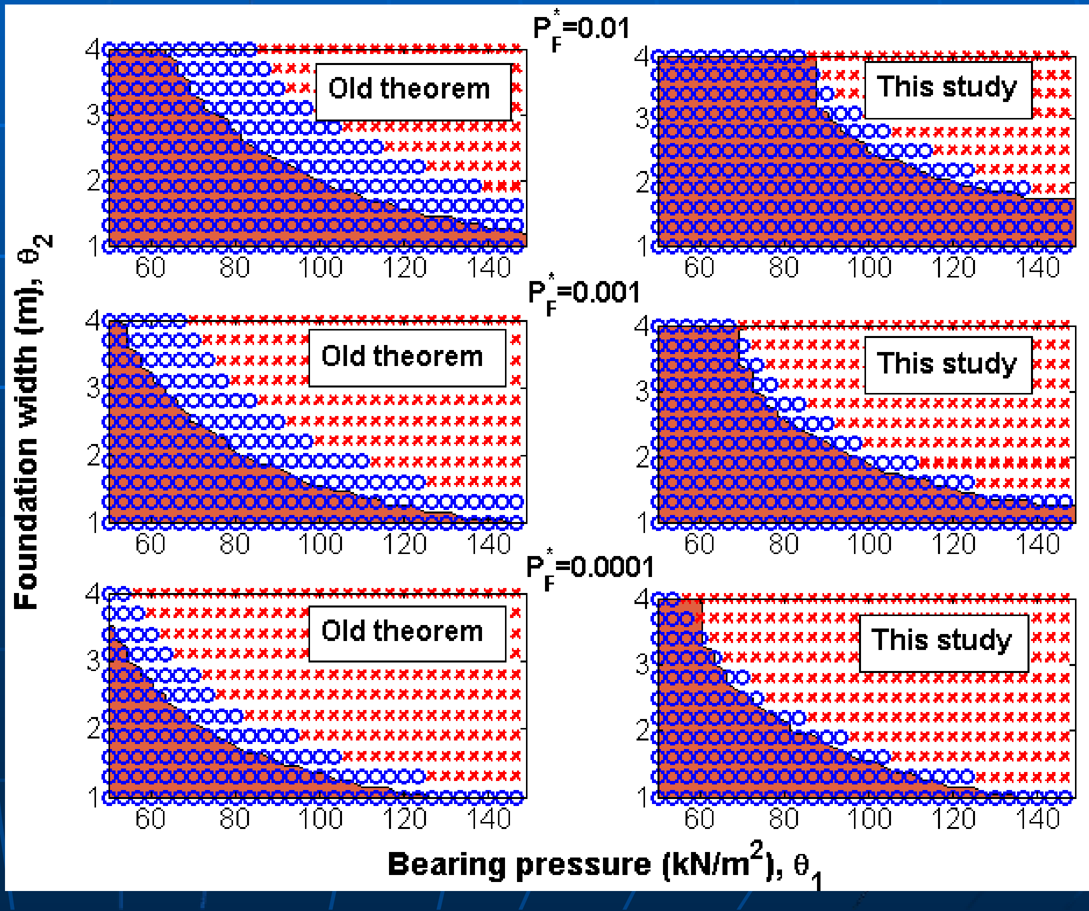
- Suppose target failure probability is 0.5^m → we require $P\left(R[Z,\theta] > 1 \middle| \theta\right) \leq 0.5^m$
- The same requirement can be transformed into $R_n(\theta) \leq 1 / \eta^*_{1-0.5^m}(\theta)$
  - Interpolation may be needed if P_F^* is not exactly 0.5^m
- Require only one set of subset simulation

Example 1

- Consolidation

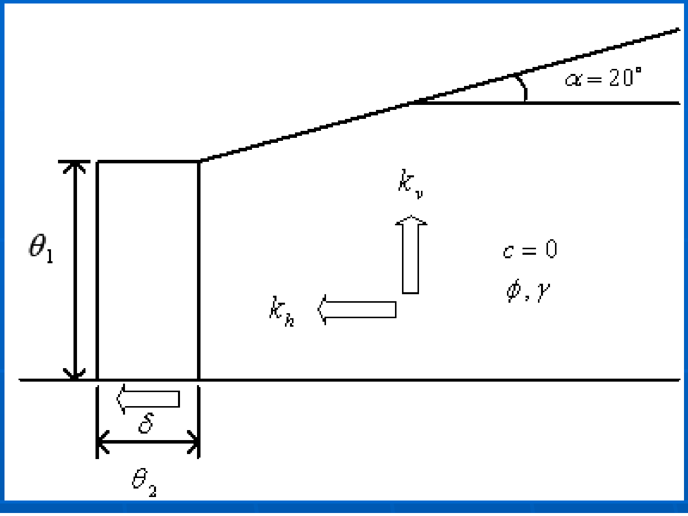


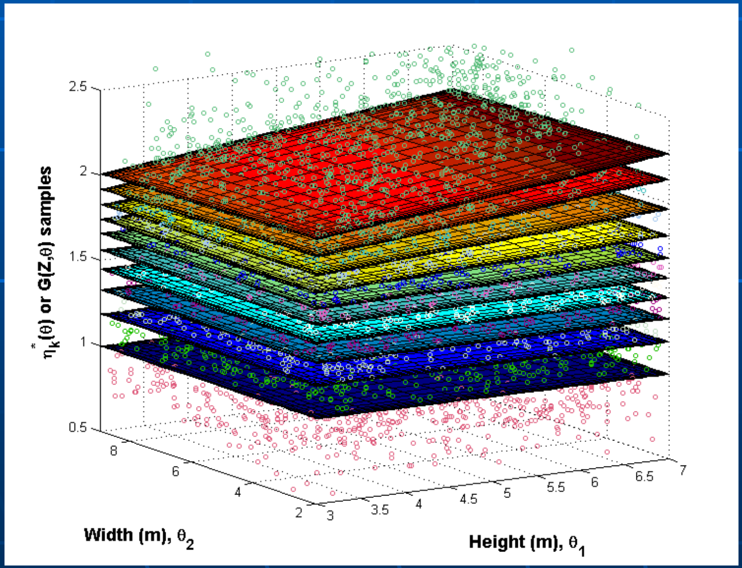


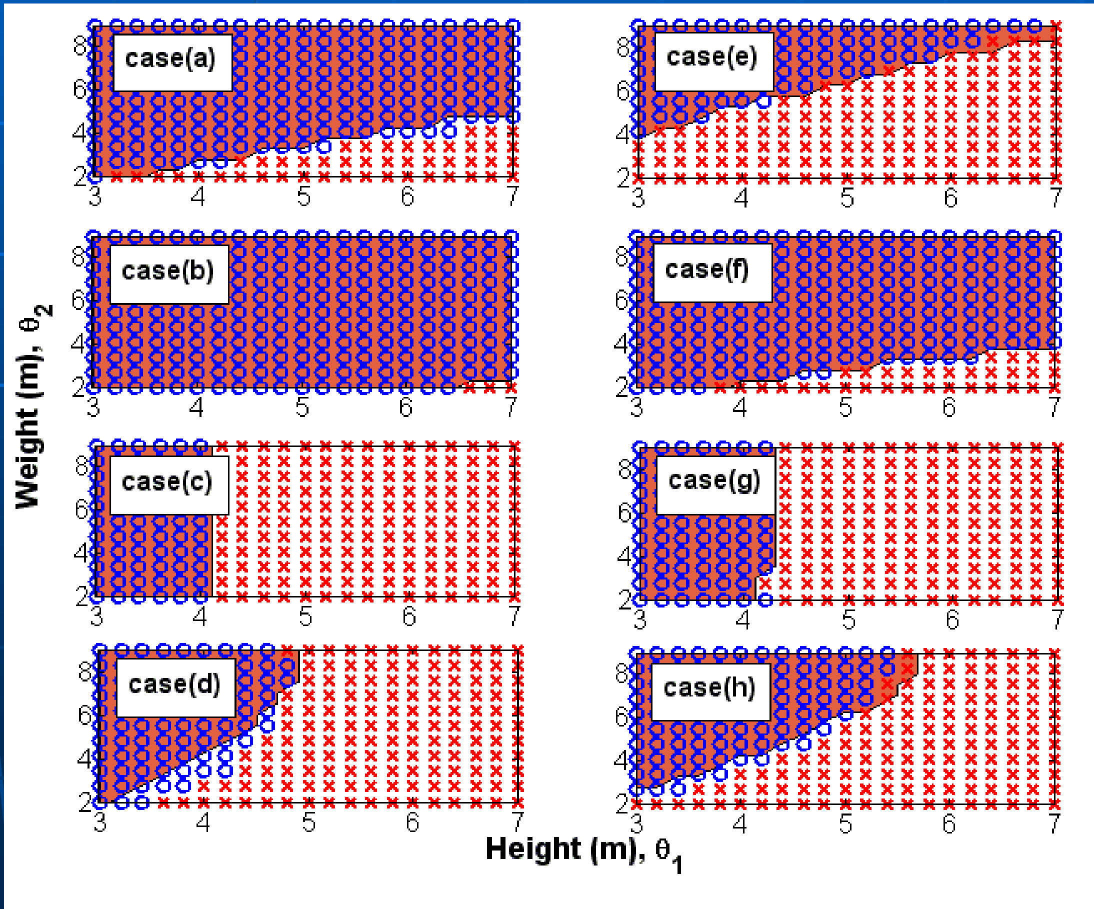


Example 2

- Retaining wall







Conclusion

- RBD can be treated as a quantile estimation problem
- It is desirable to estimate the quantile as a function of θ
- An algorithm combining subset simulation and GPD is proposed to achieve so
  - Requires only a single set of subset simulation run
- Examples showed the performance of the proposed algorithm is satisfactory

Thank you … questions?