Solving constrained optimization problems via Subset Simulation

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Objective

- To explore an extension of Subset Simulation to solving deterministic constrained optimization problem, focusing on applications in high dimensions

- Deterministic Constrained Optimization Problem

Parameters $\mathbf{x} = [x_1, \ldots, x_n] \in S \subseteq R^n$

Maximize objective function $h(\mathbf{x}) : S \mapsto R$

Subjected to constraints $g_i(\mathbf{x}) \le 0 \qquad i = 1, 2, \ldots, L$

Features/Difficulties of the Problem

- Potentially,
  - A large number of variables
  - Nonlinearity in objective function, constraint functions
  - Multiple constraints, generally nonlinear
  - Multiple local optimums may exist
  - The feasible region can have complex geometry

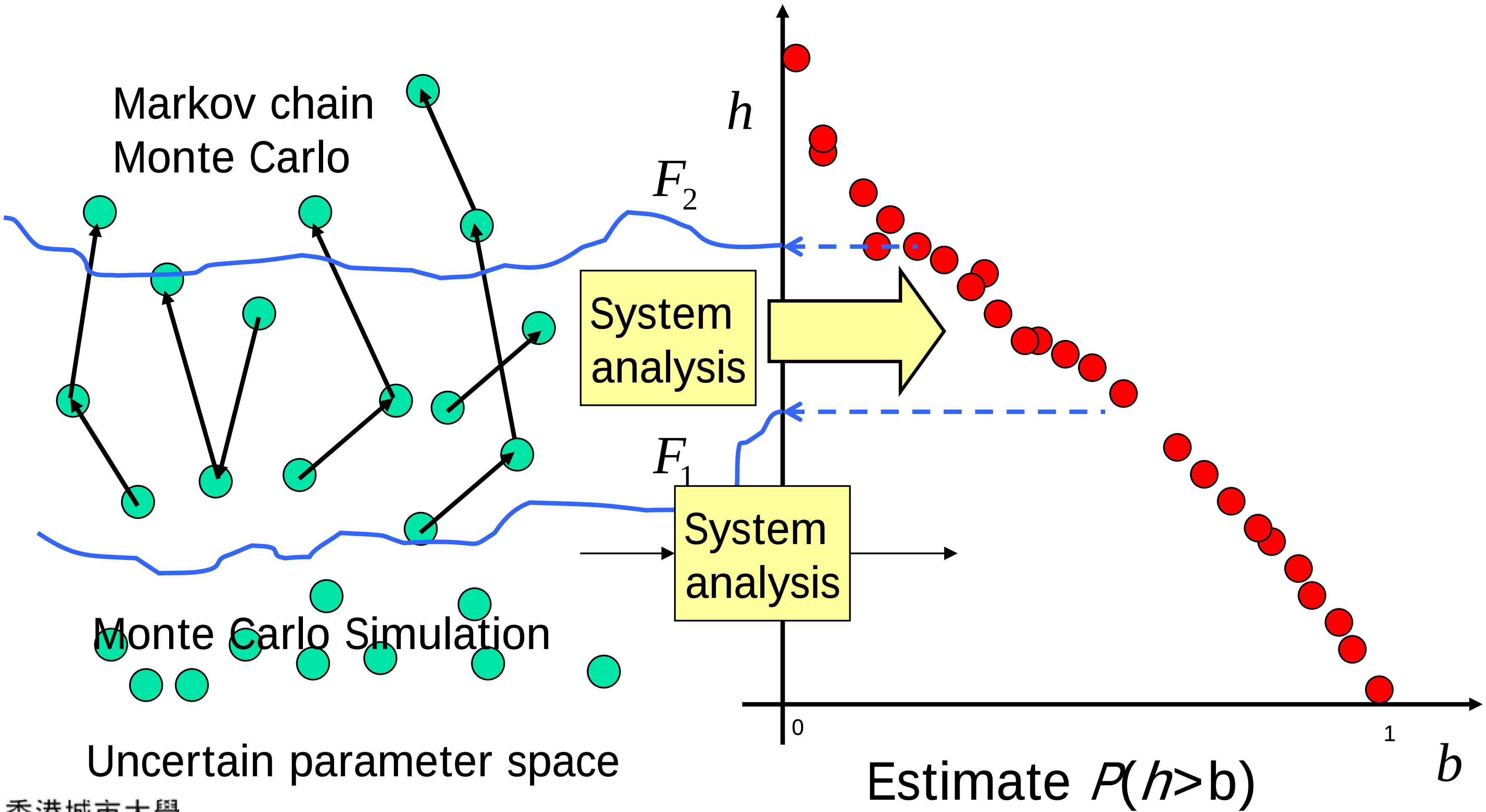
- Stochastic simulation methods may provide a robust competitive solution in this arena

Optimization vs Reliability Problem

- Explore Subset Simulation as a simulation method that has shown some robustness to dimensions and nonlinearities

- Theoretical points:
  - Augment deterministic (design) variables $\mathbf{x}$ as random variables
  - Build a link between optimization problem and reliability problem
  - Handle multiple constraints (through simulation)

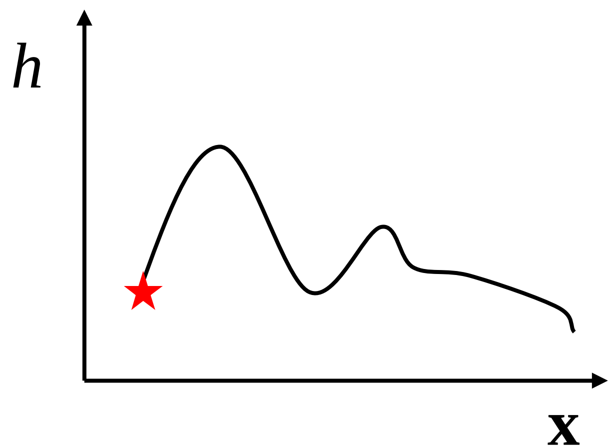
Subset Simulation (estimating $P(h(\mathbf{X}) > b)$)



Optimization vs Reliability Problem

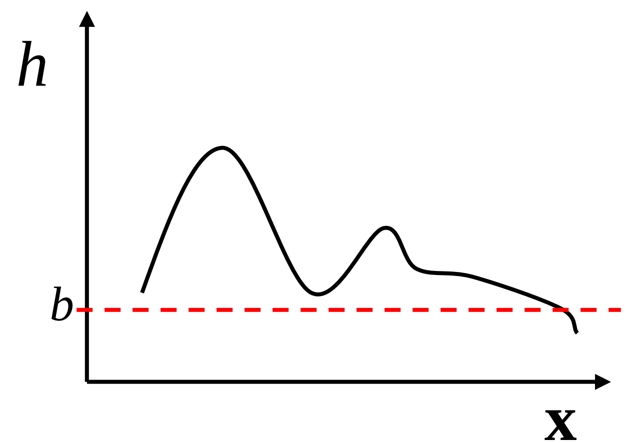
- Optimization problem
  - Design variable $\mathbf{x}$
  - Definition domain
  - Extreme event (max)

$$\text{Max. } h(\mathbf{x})$$



- Reliability problem
  - Random variable
  - Artificial distribution
  - Rare event

$$P_F = P(h(\mathbf{X}) \geq b)$$



Handling Constraints

- Incorporate constraints into the objective function
  - E.g., Lagrange multiplier, penalty function
  - Fundamental problem:
    - Scaling between h and g_i
    - Scaling among $g_1, \ldots, g_L$

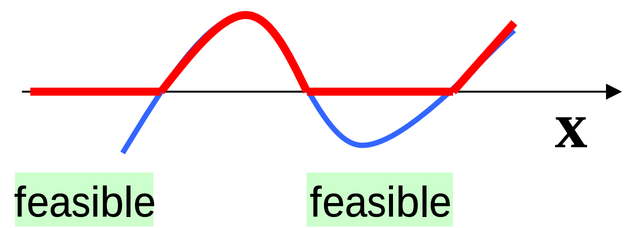
- Incorporate constraints into sample generation process
  - E.g., rejection method
  - Issue:
    - Infeasible samples are immediately rejected

- This work adopts a double-ranking criterion that attempts to keep merits of the above strategies

Double Ranking Criterion
Constraint fitness function

- For an inequality constraint $g_i(\mathbf{x}) \leq 0$, define a constraint violation function as

$$v_i(\mathbf{x}) = \begin{cases} 0 & \text{if} \quad g_i(\mathbf{x}) \leq 0 \\ g_i(\mathbf{x}) & \text{if} \quad g_i(\mathbf{x}) > 0 \end{cases}$$



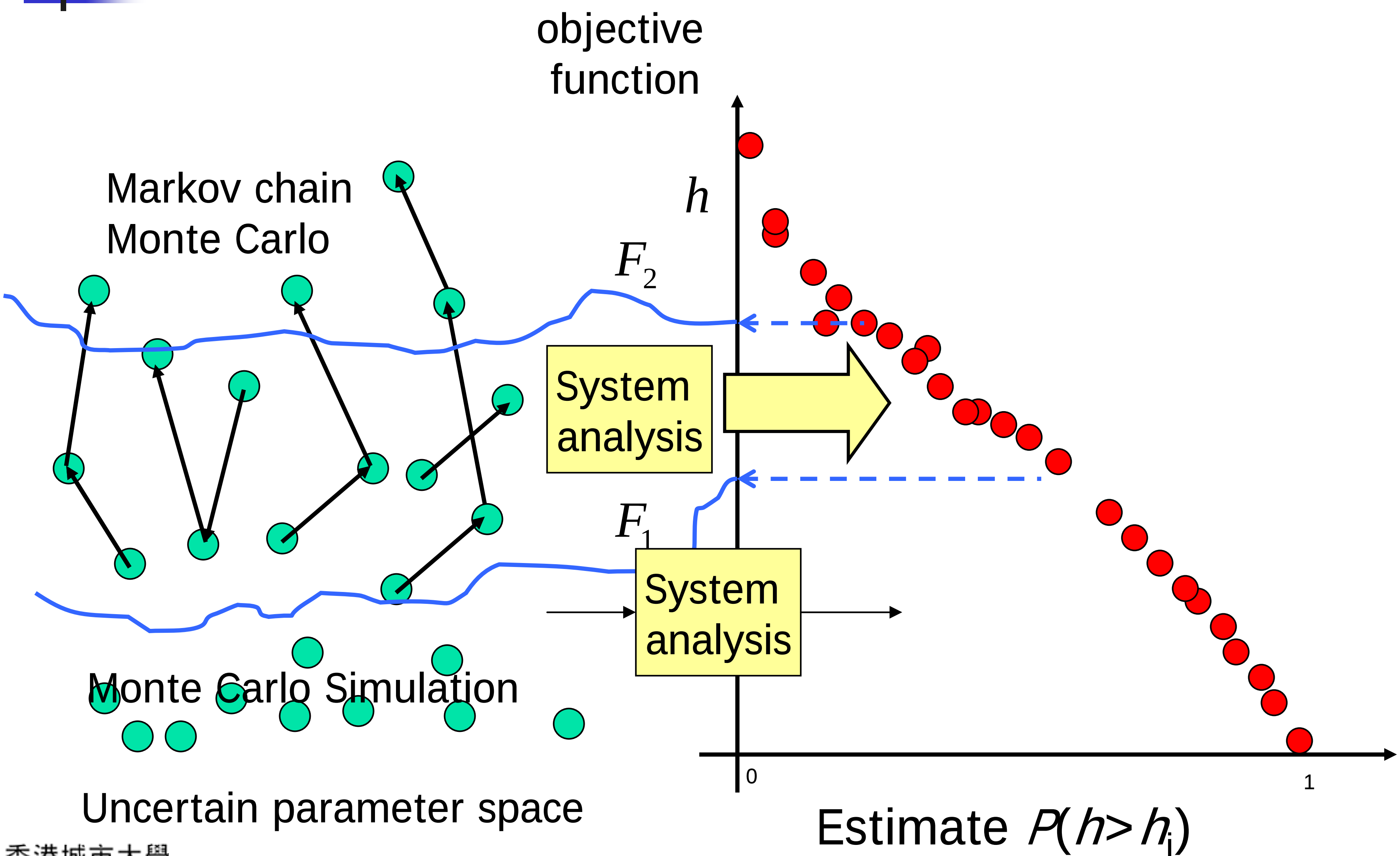
- The constraint fitness function accounts for all constraints:

$$F_{con}(\mathbf{x}) = -\max_i \ v_i$$

Note: $F_{con}(\mathbf{x}) = 0$ for feasible samples

< 0 otherwise (infeasible)

Subset Simulation



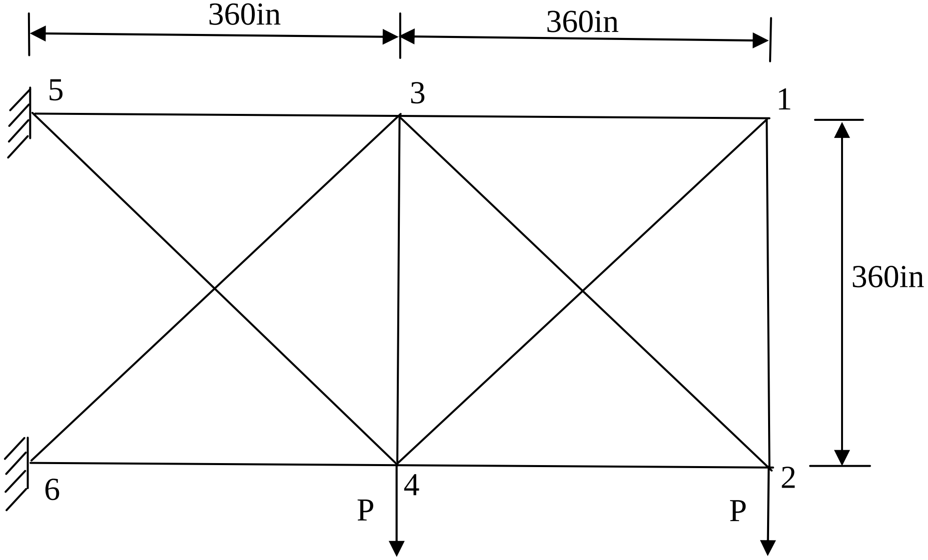
Double-Criterion Ranking

- Suppose N samples have been generated in a given simulation level
  - 1. Sort **all** samples according to F_{con}
  - 2. Sort the **feasible** samples according to h

- The ‘conditional’ samples that will be carried to the next simulation level will be taken as the top p*N samples on the list

- Note:
  - Feasible samples always have higher priority than infeasible ones
  - Not all conditional samples are feasible
  - Among the feasible samples, those with higher value of h take higher priority

Ten-bar truss design (Coello & Montes 2002)



x : 2nd moment of inertia of 10 bars

Minimize weight of truss

Constraints:

All bars: $0 < I_i < 999\,\text{in}^4$

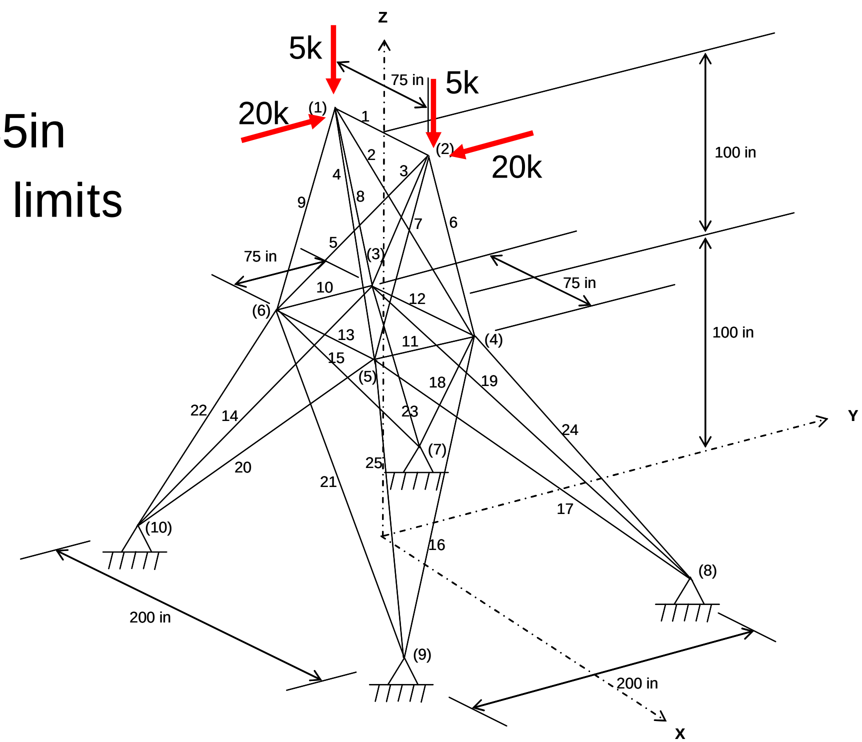
allowable stress <25ksi

All nodes: x,y disp.<2in

Application Example
Space truss design (Lee & Geem 04)

- Space truss (25 bars) design
  - Design variables: A1,...,A8 (cross-sectional areas)
  - To minimize weight
  - All 0.01<Ai<3.5 sq in
  - All nodes to have abs. x,y,z disp. <0.35in
  - All bars to have stress within specified limits

Variables		Compressive (ksi)	Tensile (ksi)
1	A_1	35.092	40.0
2	A_2–A_5	11.590	40.0
3	A_6–A_9	17.305	40.0
4	A_{10}–A_{11}	35.092	40.0
5	A_{12}–A_{13}	35.092	40.0
6	A_{14}–A_{17}	6.759	40.0
7	A_{18}–A_{21}	6.957	40.0
8	A_{22}–A_{25}	11.802	40.0



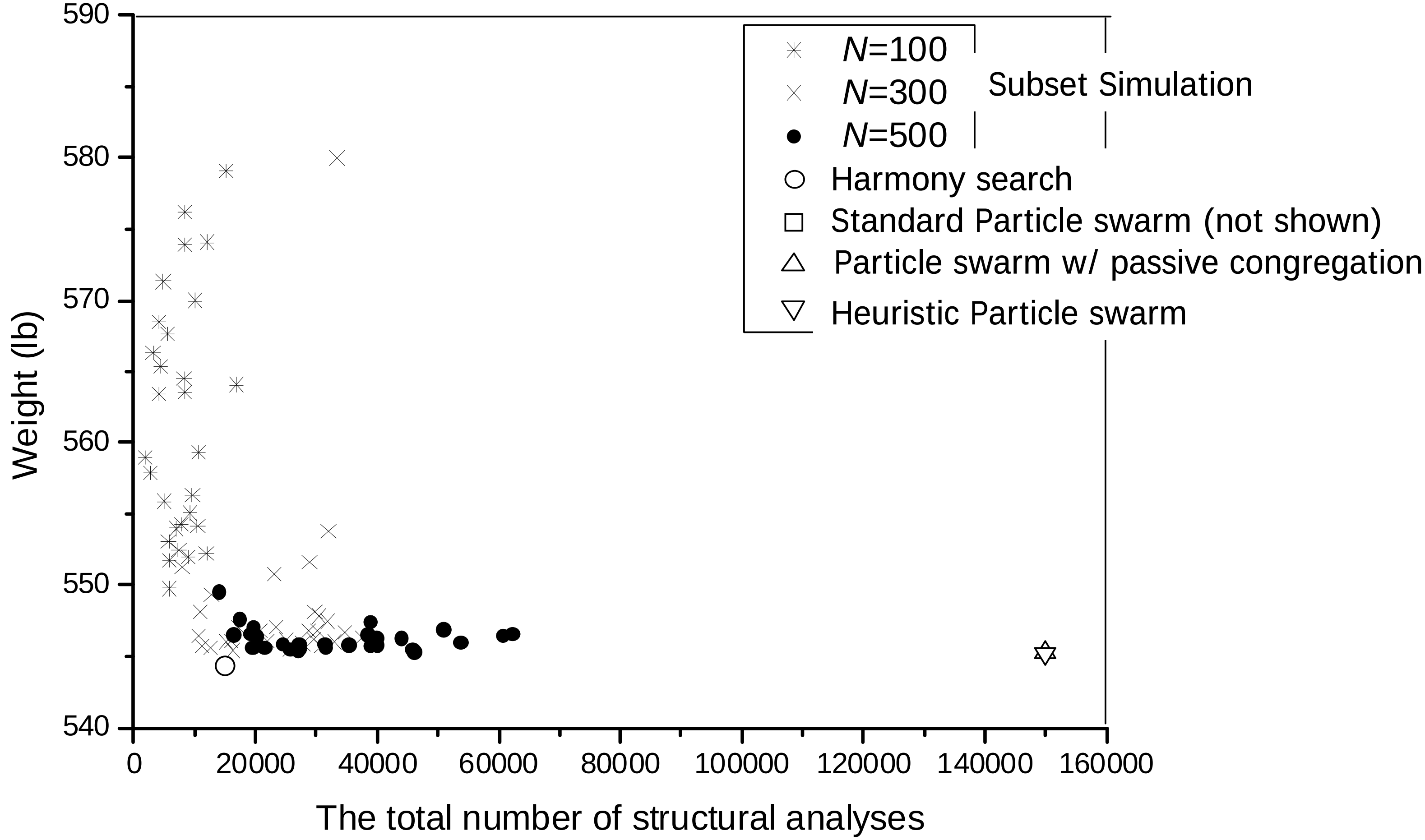
Space truss design
Results (single run)

Design Variables		Optimal cross-sectional areas (in^2)				
		Lee & Geem (2004)	Li et (2007)			SS
			PSO	PSOPC	HPSO	
1	A_1	0.047	9.863	0.010	0.010	0.010
2	A_2–A_5	2.022	1.798	1.979	1.970	2.057
3	A_6–A_9	2.950	3.654	3.011	3.016	2.892
4	A_{10}–A_{11}	0.010	0.100	0.100	0.010	0.010
5	A_{12}–A_{13}	0.014	0.100	0.100	0.010	0.014
6	A_{14}–A_{17}	0.688	0.596	0.657	0.694	0.697
7	A_{18}–A_{21}	1.657	1.659	1.678	1.681	1.666
8	A_{22}–A_{25}	2.663	2.612	2.693	2.643	2.675
Weight (lb)		544.38	627.08	545.27	545.19	545.37

Lee KS, Geem ZW (2004). A new structural optimization method based on the harmony search algorithm. Computers and Structures 2004; 82 (9-10): 781-798.

Li LJ, Huang ZB, Liu F, Wu QH (2007). A heuristic particle swarm optimizer for optimization of pin connected structures. Computers and Structures 2007; 85 (7-8): 340-349.

Space truss design
Statistical performance



Concluding remarks

- The link between optimization problem and reliability problem
  - Extreme events ~ rare events
- Double-criterion ranking method allows segregating feasible/infeasible samples and optimizing performance together.
- Outstanding issues:
  - Scaling among multiple constraints
  - Progression scheme for sample conditioning
- An extended version of this work will be published in
  - Li & Au (2010). “Design optimization using Subset Simulation algorithm”. Structural Safety, Special Issue on Modeling of Rare Events, edited by Beer, Phoon & Quek.

Acknowledgments

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Thank You