A fuzzy finite element analysis technique for structural static analysis based on interval fields

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Outline

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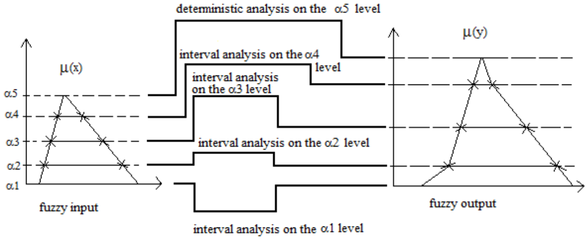
Causes of non-determinism:

- Variability
- Uncertainty
- (In)variable uncertainty and (un)certain variability

Dealing with non-determinism:

- Probabilistic approaches: probability density function (PDF)
- Non-probabilistic approaches:
  - Interval: $x^I = [x_{min}\ x_{max}] = [\underline{x}\ \overline{x}]$
  - Fuzzy: membership function $\mu_{\tilde{x}}(x)$



Field: Spatial coherence and distribution of a parameter.

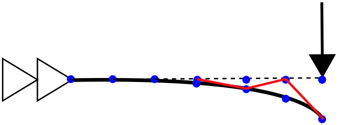
- Input side of analysis:
  - pressure distribution
  - material properties
- Output side of analysis:
  - deformation
  - temperature

In reality a strong dependency exists between the value of a parameter in one place and its value in a nearby place.
How to represent this?

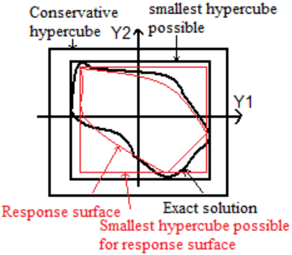
Dealing with non-deterministic fields at the output side:

- Probabilistic: Random fields, covariance function?
- Non-probabilistic: Interval vector $\{\mathbf{y}^{\mathbf{I}}\}$, dependency?

$$\{\mathbf{y}^{\mathbf{s}}\} = \{\{\mathbf{y}\} \mid (\{\mathbf{x}\} \in \{\mathbf{x}^{\mathbf{I}}\})(\{\mathbf{y}\} = f(\{\mathbf{x}\}))\}$$



Example:
interval vector representation of interval field



2D interval vector,
introducing response surface

Outline

Interval Fields

General concept

Shapefunctions to describe a spatial pattern.

$$[\psi] = [{}_{\mathbf{1}}\psi \quad {}_{\mathbf{2}}\psi \quad \ldots]$$

- Each pattern of $[\psi]$ represents a possible fixed dependency between the components of the solution field.
- The non-determinism should influence the patterns and/or their contribution to the solution as a whole.
- It is clear that a realistic set of patterns is mandatory to describe the final results.

Static finite element analysis:

$$[\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{f}\}$$

Deformation patterns for the output of a static finite element analysis:

$$[\psi]\{\mathbf{w}\} = \{\mathbf{u}\}$$

Projection of original set of equations on space of deformation patterns (classical static reduction) to find contribution of each pattern:

$$[\psi]^T[\mathbf{K}][\psi]\{\mathbf{w}\} = [\psi]^T\{\mathbf{f}\}$$

Influence of non-determinism:

Influence of variability/uncertainty $\{\mathbf{x}\} \in \{\mathbf{x^I}\}$ on:

- stiffness matrix: $[\mathbf{K}(\mathbf{x})]$
- load vector: $\{\mathbf{f}(\mathbf{x})\}$

Results in: $\{\mathbf{u}(\mathbf{x})\}$
The deformation patterns provide an intermediate spatial dependency: $[\psi(\mathbf{x})]\{\mathbf{w}\} = \{\mathbf{u}(\mathbf{x})\}$

Approximation of influence of non-determinism:

Based on a few exact evaluations, the deformation patterns are approximated in the interval space: $[\widetilde{\psi}(\mathbf{x})]\{\mathbf{v}\} = \{\widetilde{\mathbf{u}}(\mathbf{x})\}$

Objectives:

Determine optimal choice of spatial field basis at output of static structural IFE analysis:

- Based on static reduction
  Benefit: *Smaller system of equations to be solved due to the projection of the original system on a few deformation patterns*
- Using approximation for the basis in the interval space
  Benefit: *Fast approximation of the dependencies using* $[\widetilde{\psi}(\mathbf{x})]$

Criterion: accuracy of static deformation results in complete interval space

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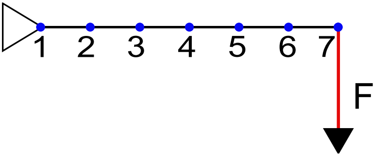
1. Introduction
2. Interval Fields
3. Method for static FE
    - Deterministic Deformation Patterns
    - Approximation of the patterns in the interval space
    - Projection on the patterns
4. Example: Vega Launcher
5. Conclusion

Method for static FE

Deterministic Deformation Patterns

The degrees of freedom are divided into subsets:

c-set	Known, constrained dofs
r-set	Dofs to be constrained to resist rigid body motion
l-set	Dofs that are loaded
o-set	Remaining dofs



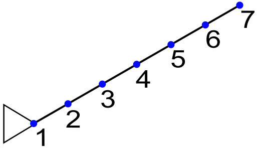
c-set = t1
r-set = r1
l-set = t7
o-set = tr2, tr3, tr4, tr5, tr6, r7

Method for static FE

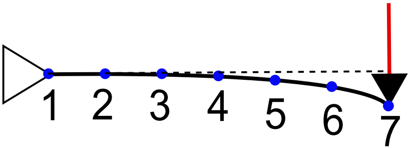
Deterministic Deformation Patterns

Commonly used deformation patterns for static analysis:

- Rigid body deformation patterns: unit displacement for r-set



- Displacement deformation patterns: unit displacement for l-set
- Force deformation patterns: unit force/moment for l-set



Method for static FE

Approximation

The deformation patterns are approximated based on nominal and deviatoric parts.

$$\{\widetilde{\psi}(\mathbf{x})\} = \{\psi_{\mathbf{nom}}\} + \{\psi_{\mathbf{dev}}\}f(\Delta x)$$

with $\Delta x = \dfrac{x - x_{nom}}{x_{nom}}$ and $f(\Delta x)$ a function with a root for $\Delta x = 0$

Possible approximations:

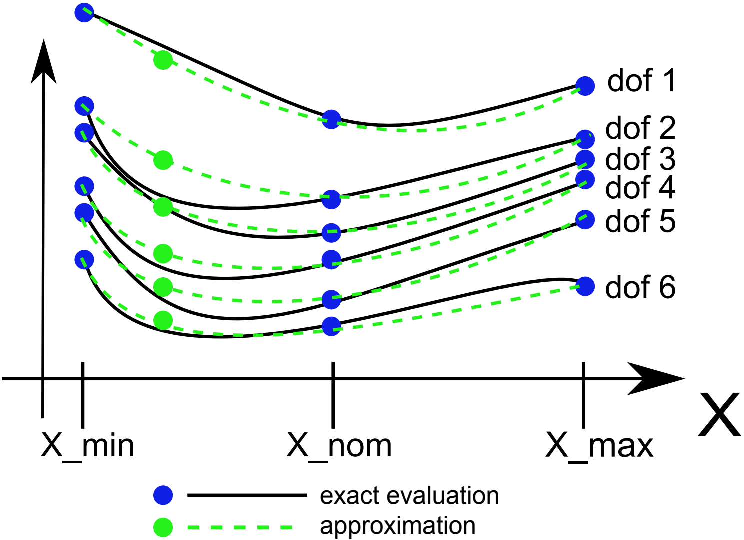
Based on three exact evaluations (nominal, maximal, minimal):

- quadratic:
  $\{\psi(\mathbf{x})\} \cong \{\widetilde{\psi}(\mathbf{x})\} = \{\psi_{\mathbf{nom}}\} + \{\psi_{\mathbf{linear}}\}\Delta x + \{\psi_{\mathbf{quadratic}}\}\Delta x^2$
- exponential:
  $\{\psi(\mathbf{x})\} \cong \{\widetilde{\psi}(\mathbf{x})\} = \{\psi_{\mathbf{nom}}\} + \{\psi_{\mathbf{amplitude}}\}(e^{\Delta x} - 1) + \{\psi_{\mathbf{linear}}\}\Delta x$

More advanced approximations can be constructed based on a higher number of exact evaluations.

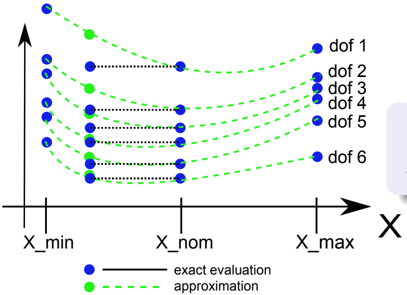
Method for static FE

Approximation example



Method for static FE

Projection



$$[\widetilde{\psi}]^T[\mathbf{K}][\widetilde{\psi}]\{\mathbf{w}\} = [\widetilde{\psi}]^T\{\mathbf{f}\}$$
$$\{\widetilde{\psi}\} = \beta\{\psi_{\mathbf{nom}}\} + \gamma\{\psi_{\mathbf{dev}}\}f(\Delta x)$$

Classic projection: ($\beta = \gamma$)

$$[\psi_{\mathbf{nom}} + \psi_{\mathbf{dev}} f(\Delta x)]^T[\mathbf{K}(\mathbf{x})][\psi_{\mathbf{nom}} + \psi_{\mathbf{dev}} f(\Delta x)]\{\mathbf{v}\} = [\psi_{\mathbf{nom}} + \psi_{\mathbf{dev}} f(\Delta x)]^T\{\mathbf{f}(\mathbf{x})\}$$

Improved projection: ($\beta \neq \gamma$)

$$[\psi_{\mathbf{nom}} \quad \psi_{\mathbf{dev}} f(\Delta x)]^T[\mathbf{K}(\mathbf{x})][\psi_{\mathbf{nom}} \quad \psi_{\mathbf{dev}} f(\Delta x)]\{\mathbf{v}\} = [\psi_{\mathbf{nom}} \quad \psi_{\mathbf{dev}} f(\Delta x)]^T\{\mathbf{f}(\mathbf{x})\}$$

Improved projection: Extensions

Multiple uncertainties/variabilities:

$$[\widetilde{\psi}(\mathbf{x})] = [\psi_{\mathbf{nom}} \quad \psi_{\mathbf{dev1}} f(\Delta x_1) \quad \psi_{\mathbf{dev2}} f(\Delta x_2) \quad \ldots]$$

Multiple dofs in I-set and multiple uncertainties/variabilities:

$$\begin{aligned}[\widetilde{\psi}(\mathbf{x})] = [& {}_{\mathbf{1}}\psi_{\mathbf{nom}} \quad {}_{\mathbf{1}}\psi_{\mathbf{dev1}} f(\Delta x_1) \quad {}_{\mathbf{1}}\psi_{\mathbf{dev2}} f(\Delta x_2) \quad \ldots \\ & {}_{\mathbf{2}}\psi_{\mathbf{nom}} \quad {}_{\mathbf{2}}\psi_{\mathbf{dev1}} f(\Delta x_1) \quad {}_{\mathbf{2}}\psi_{\mathbf{dev2}} f(\Delta x_2) \quad \ldots \\ & \ldots \quad \ldots \quad \ldots \quad \ldots]\end{aligned}$$

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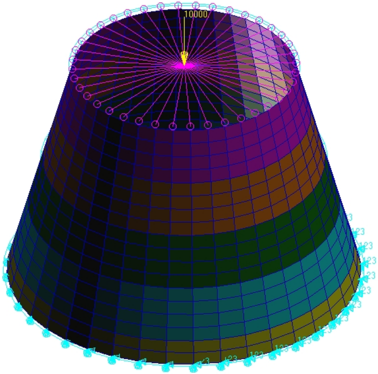
4. Example: Vega Launcher
   - Input data
   - Results
5. Conclusion

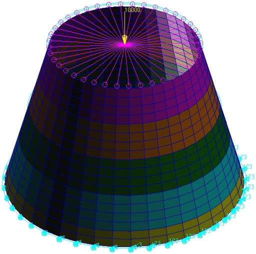
Example: Vega Launcher

Description of part

copyright *GW_Simulations*, *wikipedia*



courtesy of *DutchSpace*



Subpart of the small launcher Vega:

- DOFS: 6726
- Interval space is equidistantly sampled in 5 points, resulting in $5^5 = 3125$ cases.
- Measure of error

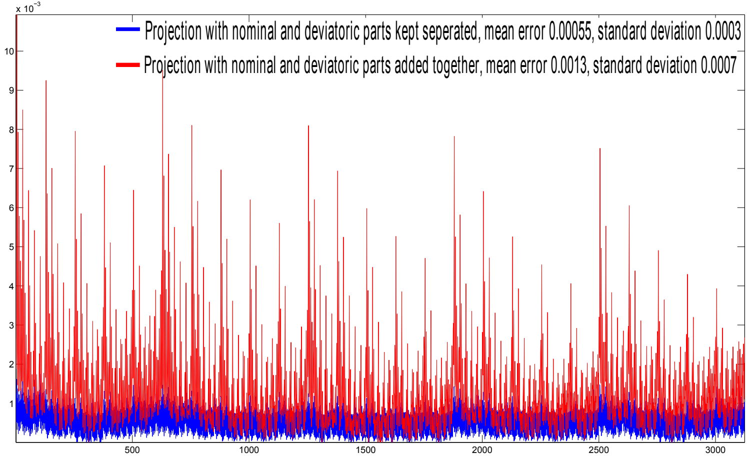
$$error = \frac{\sum_{n=1}^{n_{DOF}} \mid u_n - \tilde{u}_n \mid}{n_{DOF} u_{max}}$$

Thickness uncertainties:

Notation	Min [*mm*]	Nominal [*mm*]	Max [*mm*]
t_1	3	4	5
t_2	4	5	6
t_3	4	6	8
t_4	4	6	8
t_5	4	7	10

Example: Vega Launcher

Results



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Conclusion

- Using shape functions allows for the development of an interval field representation of uncertainties.
- Novel projection increases accuracy with little extra calculation costs.
- This representation provides a fast link between the input uncertainties and the output field for any possible interval technique.
- Extendable to any technique requiring multiple evaluations of a model influenced by non-determinism.

Questions

Questions?