Economic Emission Load Dispatch using Interval Differential Evolution

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Overview

- Introduction
- Differential Evolution (D. E.) Algorithm & its limitations
- Interval analysis & Interval subdivision algorithm
- Modified Differential Evolution using Intervals (MDEI)
- Features of MDEI & its algorithm
- Application of MDEI to Economic Emission Load Dispatch Problem (EELD)
- Description of EELD- its objectives and constraints
- Test system
- Experimental results
- Conclusions

Introduction

- Propose a new modified Differential Evolution algorithm using Intervals and is applied on Ecomonic Emission Load dispatch Problem of power system.

- Aim is to find the minimum cost of power generation subjected to system constraints for the given power demand.

Conventional methods

- Evolutionary Algorithms
- Population based meta-heuristic optimization algorithm
- Uses biological evolution
- Operators: reproduction, mutation, recombination, natural selection
- Examples: GA , EP, ES, DE etc.

Differential Evolution (D. E.)

- A simple and efficient direct search algorithm.
- Introduced by R. Storn & K. Prince in 1994.
- Uses operators : crossover, mutation and selection.
- Relies on mutation operation, used as search mechanism.
- Terminates with maximum no. of iterations (or function evaluations) or user defined minimum value.

Limitations of the D. E.

- Global minima is not guaranteed with less no. of iterations.
- Wastage of computational effort with excessive no. of iterations.
- Estimate of the global minimum is not available for real world problems.
- Slow convergence near region of global minimum

Modified D. E. using Intervals (MDEI)

- Differential Evolution
- Interval analysis - interval subdivision algorithm
- Hybridization of Differential Evolution with Interval Analysis

D.E. Algorithm Parameters

- Input parameters
  - Dimension of the problem
  - Lower and upper boundaries of domain
- Controlling parameters
  - Population size $NP \in [20, 50]$
  - Scaling factor $\mu \in [0,1]$
  - Crossover constant $CR \in [0,1]$
  - Max no. of function evaluations
- Output Parameters
  - Global minimum
  - Minimizer

D. E. Optimization Process

- The population of constant size is initialized randomly.
- A mutant vector generated using scaling factor and population members.
- Crossover generates a trial vector, combination of a mutant and parent vector based on probability distributions using cossover constant.
- Selection directs the search toward the prospective regions.

D.E. Algorithm

- Population initialization
- Evaluation of population
- Find the vector with the lowest function value
- While the termination criterion not reached do

    Mutation

    Crossover

    Selection

    Evaluation of new population.
- Output Global minimum & Minimizer

Features of Interval Analysis

- Developed by R. Moore in1966.
- Real numbers replaced by real intervals.
- Real arithmetic is replaced by Interval arithmetic.
- Provides a tool for estimating and controlling errors like - rounding, approximations, and uncertain data, automatically.
- Enables to compute the bounds on the true answer of prescribed accuracy.
- Provides global information over wide interval – bounds on the ranges .

The Interval

- A real interval

$$\mathrm{x} = \{[a,b] \,|\, a \le b, a, b \in R\}$$

a is infimum & b is supremum

- Width of the interval

$$w(\mathrm{x}) = b - a$$

- Midpoint of Interval

$$m(\mathrm{x}) = (a+b)/2$$

Interval Extension of Expression

- A natural interval extension of f on the box $\mathbf{x}$ is obtained by replacing, in the expression for f,

  - all occurrences of real x_i with intervals $\mathbf{x}_i$
  - and all real operations with the corresponding interval operations.

Interval Subdivision Algorithm

- Interval Analysis based global optimization.
- Successive domain reduction approach.
- Accelerating techniques to discard the boxes : Cut off test, Monotonicity test.
- Gives the list of small boxes containing maxima, minima or saddle points.
- Terminates with user defined domain width or function width in list.
- Convergence is guaranteed

Accelerating Device: Cut Off Test

- Uses interval extension function of current subbox $F(\mathrm{x})$ and the current upper bound $\overline{f}$ for global minimum.
- The subbox with $\inf(F(\mathrm{x})) > \overline{f}$ is discarded.

Accelerating Device: Monotonicity test

- Uses the interval extension of gradient of $f(\mathrm{x})$ of current subbox $\nabla F(\mathrm{x})$.

- The box with $0 \notin \nabla F(\mathrm{x})$ is discarded.

Interval Subdivision Algorithm parameters

- Inputs

  The initial box $\mathbf{x}$

  The objective function f

  The maximum diameter of the box ε to start the DE algorithm

- Outputs

  The list of boxes L

  The interval F^* containing the initial bounds for global minimum f^*

Interval Subdivision Algorithm

- Initialize working list consisting of initial box, lower bound and candidate list

- While the termination criterion not reached do
  - Bisect the box parallel to the max width

  - Form interval extension functions for subboxes

  - Update the function upper bound & hence the working list
  - Perform Cut off test

Interval Subdivision Algorithm (Cont.)

- Perform Monotonocity test
- Update the candidate list
- Select the interval for subdivision

- Output

The candidate list L

The interval F^* containing the initial bounds for global minimum f^*

Features of MDEI

- Estimation of bounds using interval arithmetic
- Initialization of population using interval subdivision approach $F^* = [\underline{F}^*, \overline{F}^*]$
- Population modification after mutation, crossover, and selection using subset of current population.
- Updating bounds using shrinking box constructed with the subset
- Converges with reduced width of shrinking box
- New termination criterion using shrinking box width

Population Initialization

- Define population size NP
- Population initialized with midpoints of the boxes in the candidate list L

- If the no. of boxes is greater than NP, population size NP is replaced by no. of boxes.

- If the no. of boxes is less than NP
  - Select box in sequence.
  - Select a point with in the box randomly.
  - Add the point to the population.

Population Modification

- Population is modified for next generation using the subset S of current population with k no. of members.

- Subset S contains members with function values in F^*

- Population Modification
  - Population is replaced by S
  - Form a population of NP size by selecting a point randomly with in S and add the point to the population.

Termination Criterion

- Shrinking box x_S is a interval vector contains subset S
- It is constructed with $k > r$ where r is the user defined shrinking constant.
- Bounds $\underline{F_S}$ & $\overline{F_S}$ are obtained for x_S
- Current bounds updated

$$\overline{F^*} = min\,(\overline{F^*}, \overline{F_S}) \qquad \underline{F^*} = max\,(\underline{F^*}, \underline{F_S})$$

- Terminates with $w(\mathrm{x}_S) < \varepsilon_x$ & $w(F) < \varepsilon_f$

MDEI Paramaters

Input parameters

- The initial box $\mathbf{x}$
- The objective function f
- The maximum diameter of the box ε
- Tolerances on box width ε_x and function width ε_f

Output parameters

- Global minimum f^*
- Minimizer $\mathbf{x}^*$

Controlling parameters

- Shrinking constant $r < NP$
- Population size $NP \in [20,50]$
- Scaling factor $\mu \in [0,1]$
- Crossover constant $CR \in [0,1]$

MDEI Algorithm

- Apply interval subdivision algorithm.
- Population initialization with NP
- Initialization of bounds
- Evaluation of population.
- While the termination criterion not reached do
    - Mutation
    - Crossover
    - Selection
    - Modify the population.
    - Update the bounds.
    - Evaluation of new population.
- Output f^* and x^*

Application of MDEI and DE to Economic Emission Load Dispatch Problem (EELD)

- Both algorithms DE and MDEI implemented using MATLAB 6.1

- Performance comparison with EELD problem

Economic Emission Load Dispatch (EELD)

"To figure out the optimal amount of generated power for the fossil based generating unit in the system by minimizing the fuel cost and emission cost simultaneously subject to various equality and inequality constraints"

EELD Objectives

- Fuel cost:

$$C(P) = \sum_{i=1}^{N} a_i + b_i P_i + c_i P_i^{\,2}$$

N is the no. of gen units

a_i, b_i, c_i are cost coefficients of i^{th} gen

P_i is power output Of i^{th} gen

P is the vector of real power outputs of generators

- Emission cost:

$$E(P) = \sum_{i=1}^{N} \alpha_i + \beta_i P_i + \gamma_i P_i^2 + d_i \exp(e_i P_i)$$

$\alpha_i, \beta_i, \gamma_i, d_i, e_i$ are cost coefficients of i^{th} gen

Total EELD Objective

$$F(P) = \delta C(P) + (1 - \delta) k E(P)$$

k is emission control cost factor obtained from test system

$\delta \in [0,1]$ is a weighting constant

EELD Constraints

- Equality constraint

$$\sum_{i=1}^{N} P_i = P_D + P_{loss}$$

P_D Power demand & P_{loss} Power loss

- Inequality constraint

$$P_i^{\min} \leq P_i \leq P_i^{\max}$$

$P_i^{\min}$ & $P_i^{\max}$ are the minimum and maximum power output of i^{th} generator

Formulation of EELD Problem

EELD is formulated as

$$minimize F(P)$$

Subject to

$$g(P) = 0$$

$$h(P) \leq 0$$

Test System

- The lossless IEEE 30-bus 6-generator system
- The p.u. generation capacity for all generating units $P^{\min} = 0.5$ & $P^{\max} = 1.5$
- Power demand $P_D = 2.84\,p.u.$
- Emission control cost factor $k = 30.0738$
- Two cases $\delta = 0$ & $\delta = 1$

Fuel cost & Emission Coefficients

Unit	a_i	b_i	c_i	α_i	β_i	γ_i	d_i	e_i
1	**10**	**200**	**100**	**4.091**	**5.554**	**6.490**	**2.0E - 4**	**2.857**
2	**10**	**150**	**120**	**2.543**	**6.047**	**5.638**	**5.0E - 4**	**3.333**
3	**20**	**180**	**40**	**4.258**	**5.094**	**4.586**	**1.0E - 6**	**8.000**
4	**10**	**100**	**60**	**5.326**	**3.550**	**3.380**	**2.0E - 3**	**2.000**
5	**20**	**180**	**40**	**4.258**	**5.094**	**4.586**	**1.0E- 6**	**8.000**
6	**10**	**150**	**100**	**6.131**	**5.555**	**5.151**	**1.0E- 5**	**6.667**

Experimental data (DE and MDEI)

- The maximum diameter of the box $\varepsilon = 0.7$
- Tolerances $\varepsilon_x = 10^{-7}$ and $\varepsilon_f = 10^{-3}$
- Shrinking constant $r = 5$
- Population size $NP = 50$
- Scaling factor $\mu = 0.95$
- Crossover constant $CR = 0.8$
- Max no. of function evaluations obtained from MDEI
- 20 runs

Experimental Results for Case I

Algorithm	BV	NS	ENES	MNE
DE	600.23	-	-	9868
MDEI	600.11	20	5950	6190

- **BV**- Best value
- **NS**- Number of successful runs
- **ENES**- Mean value of function evaluation
- **MNE**- Max no. of real function evaluation for a particular run

Experimental Results for Case II

Algorithm	BV	NS	ENES	MNE
DE	572.1115	-	-	11278
MDEI	572.1112	20	6578	7600

- **BV**- Best value
- **NS**- Number of successful runs
- **ENES**- Mean value of function evaluation
- **MNE**- Max no. of real function evaluation for a particular run

Optimum Power Outputs using MDEI

Gen unit	CaseI (p. u.)	CaseII (p.u.)
P1	0.11	0.3904
P2	0.30	0.4932
P3	0.524	0.5025
P4	1.016	0.4533
P5	0.524	0.5024
P6	0.360	0.4921

Conclusions

- Conventional DE has poor termination criterion.
- Inclusion of interval arithmetic in conventional DE (MDEI)
- Highly performing initial population formed using branch & bound principle
- Bounds are updated at each generation.
- Termination criterion on the concept of shrinking box
- The global minimum obtained with certainty with reduced computational effort
- Applied to EELD problem of IEEE 30 bus 6 generators system
- Reults are compared with Conventional DE
- MDEI outperforms conventional DE

Thank You