

Model Fusion under Probabilistic and Interval Uncertainty, with Application to Earth Sciences

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1. Need to Combine Data from Different Sources

- In many areas of science and engineering, we have different sources of data.
- For example, in geophysics, there are many sources of data for Earth models:
 - first-arrival passive seismic data (from the actual earthquakes);
 - first-arrival active seismic data (from the seismic experiments);
 - gravity data; and
 - surface waves.

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2. Need to Combine Data (cont-d)

- Datasets coming from different sources provide complementary information.
- *Example:* different geophysical datasets contain different information on earth structure.
- In general:
 - some of the datasets provide better accuracy and/or spatial resolution in some spatial areas;
 - other datasets provide a better accuracy and/or spatial resolution in other areas or depths.
- *Example:*
 - gravity measurements have (relatively) low spatial resolution;
 - a seismic data point comes from a narrow trajectory of a seismic signal – so spatial resolution is higher.

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3. Joint Inversion: An Ideal Future Approach

- *At present:* each of the datasets is often processed separately.
- *It is desirable:* to combine data from different datasets.
- *Ideal approach:* use all the datasets to produce a single model.
- *Problem:* in many areas, there are no efficient algorithms for simultaneously processing all the datasets.
- *Challenge:* designing joint inversion techniques is an important theoretical and practical challenge.

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4. Proposed Solution – Model Fusion: Main Idea

- *Reminder*: joint inversion methods are still being developed.
- *Practical solution*: to fuse the *models* coming from different datasets.
- *Simplest case – data fusion, probabilistic uncertainty*:
 - we have several estimates $\tilde{x}^{(1)}, \dots, \tilde{x}^{(n)}$ of the same quantity x .
 - each estimation error $\Delta x^{(i)} \stackrel{\text{def}}{=} \tilde{x}^{(i)} - x$ is normally distributed with 0 mean and known st. dev. $\sigma^{(i)}$;
 - Least Squares: find x that minimizes $\sum_{i=1}^n \frac{(\tilde{x}^{(i)} - x)^2}{2 \cdot (\sigma^{(i)})^2}$;

– solution:
$$x = \frac{\sum_{i=1}^n \tilde{x}^{(i)} \cdot (\sigma^{(i)})^{-2}}{\sum_{i=1}^n (\sigma^{(i)})^{-2}}.$$

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5. Data Fusion: Case of Interval Uncertainty

- In some practical situations, the value x is known with interval uncertainty.
- This happens, e.g., when we only know the upper bound $\Delta^{(i)}$ on each estimation error $\Delta x^{(i)}$: $|\Delta x^{(i)}| \leq \Delta_i$.
- In this case, we can conclude that $|x - \tilde{x}^{(i)}| \leq \Delta^{(i)}$, i.e., that $x \in \mathbf{x}^{(i)} \stackrel{\text{def}}{=} [\tilde{x}^{(i)} - \Delta^{(i)}, \tilde{x}^{(i)} + \Delta^{(i)}]$.
- Based on each estimate $\tilde{x}^{(i)}$, we know that the actual value x belongs to the interval $\mathbf{x}^{(i)}$.
- Thus, we know that the (unknown) actual value x belongs to the intersection of these intervals:

$$\mathbf{x} \stackrel{\text{def}}{=} \bigcap_{i=1}^n \mathbf{x}^{(i)} = [\max(\tilde{x}^{(i)} - \Delta^{(i)}), \min(\tilde{x}^{(i)} + \Delta^{(i)})].$$

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6. Additional Problem: We Also Have Different Spatial Resolution

- In many situations, different models have not only different accuracy, but also different spatial resolution.
- *Example:*
 - seismic data leads to higher spatial resolution estimates of the density at different locations, while
 - gravity data leads to lower-spatial resolution estimates of the same densities.
- *Towards precise formulation of the problem:*
 - High spatial resolution estimates correspond to small spatial cells.
 - A low spatial resolution estimate is affected by several neighboring spatial cells.

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7. Towards Formulation of a Problem

- *What is given:*
 - we have high spatial resolution estimates $\tilde{x}_1, \dots, \tilde{x}_n$ of the values x_1, \dots, x_n in several small cells;
 - we also have low spatial resolution estimates \tilde{X}_j for the weighted averages

$$X_j = \sum_{i=1}^n w_{j,i} \cdot x_i.$$

- *Objective:* based on the estimates \tilde{x}_i and \tilde{X}_j , we must provide more accurate estimates for x_i .
- *Geophysical example:* we are interested in the densities x_i .

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8. Model Fusion: Case of Probabilistic Uncertainty

We take into account several different types of approximate equalities:

- Each high spatial resolution value \tilde{x}_i is approximately equal to the actual value x_i , w/known accuracy $\sigma_{h,i}$:

$$\tilde{x}_i \approx x_i.$$

- Each lower spatial resolution value \tilde{X}_j is approximately equal to the weighted average, w/known accuracy $\sigma_{l,j}$:

$$\tilde{X}_j \approx \sum_i w_{j,i} \cdot x_i.$$

- We usually have a prior knowledge $x_{pr,i}$ of the values x_i , with accuracy $\sigma_{pr,i}$: $x_i \approx x_{pr,i}$.
- Also, each lower spatial resolution value \tilde{X}_j is \approx the value within each of the smaller cells:

$$\tilde{X}_j \approx x_{i(l,j)}.$$

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9. Case of Probabilistic Uncertainty: Details

- Each lower spatial resolution value \tilde{X}_j is approximately equal to the value within each of the smaller cells:

$$\tilde{X}_j \approx x_{i(l,j)}.$$

- The accuracy of $\tilde{X}_j \approx x_{i(l,j)}$ corresponds to the (empirical) standard deviation:

$$\sigma_{e,j}^2 \stackrel{\text{def}}{=} \frac{1}{k_j} \cdot \sum_{l=1}^{k_j} (\tilde{x}_{i(l,j)} - E_j)^2,$$

where

$$E_j \stackrel{\text{def}}{=} \frac{1}{k_j} \cdot \sum_{l=1}^{k_j} \tilde{x}_{i(l,j)}.$$

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10. Model Fusion: Least Squares Approach

- *Main idea:* use the Least Squares technique to combine the approximate equalities.
- We find the desired combined values x_i by minimizing the corresponding sum of weighted squared differences:

$$\sum_{i=1}^n \frac{(x_i - \tilde{x}_i)^2}{\sigma_{h,i}^2} + \sum_{j=1}^m \frac{1}{\sigma_{l,j}^2} \cdot \left(\tilde{X}_j - \sum_{i=1}^n w_{j,i} \cdot x_i \right)^2 +$$
$$\sum_{i=1}^n \frac{(x_i - x_{pr,i})^2}{\sigma_{pr,i}^2} + \sum_{j=1}^m \sum_{l=1}^{k_j} \frac{(\tilde{X}_j - x_{i(l,j)})^2}{\sigma_{e,j}^2}.$$

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11. Model Fusion: Solution

- To find a minimum of an expression, we:
 - differentiate it with respect to the unknowns, and
 - equate derivatives to 0.
- Differentiation with respect to x_i leads to the following system of linear equations:

$$\frac{1}{\sigma_{h,i}^2} \cdot (x_i - \tilde{x}_i) + \sum_{j:j \ni i} \frac{1}{\sigma_{l,j}^2} \cdot w_{j,i} \cdot \left(\sum_{i'=1}^n w_{j,i'} \cdot x_{i'} - \tilde{X}_j \right) + \frac{1}{\sigma_{pr,i}^2} \cdot (x_i - x_{pr,i}) + \sum_{j:j \ni i} \frac{1}{\sigma_{e,j}^2} \cdot (x_i - \tilde{X}_j) = 0,$$

where $j \ni i$ means that the j -th low spatial resolution estimate covers i -th cell.

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12. Simplification: Fusing High Spatial Resolution Estimates and Prior Estimates

- *Idea:* fuse each high spatial resolution estimate \tilde{x}_i with a prior estimate $x_{pr,i}$.
- *Detail:* instead of $\frac{1}{\sigma_{h,i}^2} \cdot (x_i - \tilde{x}_i) + \frac{1}{\sigma_{pr,i}^2} \cdot (x_i - x_{pr,i})$, we have a single term $\sigma_{f,i}^{-2} \cdot (x_i - x_{f,i})$, where

$$x_{f,i} \stackrel{\text{def}}{=} \frac{\tilde{x}_i \cdot \sigma_{h,i}^{-2} + x_{pr,i} \cdot \sigma_{pr,i}^{-2}}{\sigma_{h,i}^{-2} + \sigma_{pr,i}^{-2}}, \quad \sigma_{f,i}^{-2} \stackrel{\text{def}}{=} \sigma_{h,i}^{-2} + \sigma_{pr,i}^{-2}.$$

- *Resulting simplified equations:*

$$\sigma_{f,i}^{-2} \cdot (x_i - x_{f,i}) + \sum_{j:j \ni i} \frac{1}{\sigma_{l,j}^2} \cdot w_{j,i} \cdot \left(\sum_{i'=1}^n w_{j,i'} \cdot x_{i'} - \tilde{X}_j \right) + \sum_{j:j \ni i} \frac{1}{\sigma_{e,j}^2} \cdot (x_i - \tilde{X}_j) = 0.$$

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13. Case of a Single Low Spatial Resolution Estimate

- *Simplest case:* we have exactly one low spatial resolution estimate \tilde{X}_1 .
- *In general:* we only have high spatial resolution estimates for *some* of the cells.
- *In geosciences:* such a situation is typical: e.g.,
 - we have a low spatial resolution gravity estimates which cover a huge area in depth, and
 - we have high spatial resolution seismic estimates which only cover depths above the Moho.
- *For convenience:* let us number the cells for which we have high spatial resolution estimates first.
- Let h denote the total number of such cells.

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14. Case of a Single Low Spatial Resolution Estimate: Simplified Algorithm

First, we compute the auxiliary value

$$\mu \stackrel{\text{def}}{=} \frac{1}{\sigma_{l,1}^2} \cdot \left(\sum_{i'} w_{1,i'} \cdot x_{i'} - \tilde{X}_1 \right)$$

as $\mu = \frac{N}{D}$, where

$$N = \sum_{i=1}^h \frac{w_{1,i} \cdot (x_{f,i} - \tilde{X}_1)}{1 + \frac{\sigma_{f,i}^2}{\sigma_{e,1}^2}},$$

and

$$D = \sigma_{l,1}^2 + \sum_{i=1}^h \frac{w_{1,i}^2 \cdot \sigma_{f,i}^2}{1 + \frac{\sigma_{f,i}^2}{\sigma_{e,1}^2}} + \left(\sum_{i=h+1}^n w_{1,i}^2 \right) \cdot \sigma_{e,1}^2.$$

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15. Case of a Single Low Spatial Resolution Estimate: Simplified Algorithm (cont-d)

- Once we know μ , we compute the desired estimates for x_i , $i = 1, \dots, h$, as

$$x_i = \frac{x_{f,i}}{1 + \frac{\sigma_{f,i}^2}{\sigma_{e,1}^2}} - \frac{w_{1,i} \cdot \sigma_{f,i}^2}{1 + \frac{\sigma_{f,i}^2}{\sigma_{e,1}^2}} \cdot \mu + \tilde{X}_1 \cdot \frac{\frac{\sigma_{f,i}^2}{\sigma_{e,1}^2}}{1 + \frac{\sigma_{f,i}^2}{\sigma_{e,1}^2}}.$$

- We also compute estimates x_i for $i = h + 1, \dots, n$, as

$$x_i = \tilde{X}_1 - w_{1,i} \cdot \sigma_{e,1}^2 \cdot \mu.$$

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16. Numerical Example: Description

- *Objective:* to illustrate the above formulas.
- *Idea:* consider the simplest possible case, when we have
 - exactly one low spatial resolution estimate \tilde{X}_1
 - that covers all n cells,

and when:

- all the weights are all equal $w_{1,i} = 1/n$;
- there is a high spatial resolution estimate corresponding to each cell ($h = n$);
- all high spatial resolution estimates have the same accuracy $\sigma_{h,i} = \sigma_h$;
- $\sigma_{l,1} \ll \sigma_h$, so $\sigma_{l,1} \approx 0$; and
- there is no prior information, so $\sigma_{pr,i} = \infty$ and thus, $x_{f,i} = \tilde{x}_i$ and $\sigma_{f,i} = \sigma_h$.

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17. Additional Simplification

- *In general:* there are cells for which there are no high spatial resolution estimates.
- *How to deal with these cells:* we added a heuristic rule that
 - each lower spatial resolution value is approximately equal to the value within each of the constituent cells,
 - with the accuracy corresponding to the (empirical) standard deviation $\sigma_{e,j}$.
- *In our simplified example:* we have high spatial resolution estimate in each cell.
- So, there is no need for this heuristic rule.
- The corresponding heuristic terms in the least squares approach are proportional to $\frac{1}{\sigma_{e,1}^2}$, so we take $\sigma_{e,1}^2 = \infty$.

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18. Formulas for the Simplified Case and Numerical Example

- *Resulting formulas:* $x_i = \tilde{x}_i - \lambda$, where

$$\lambda \stackrel{\text{def}}{=} \frac{1}{n} \cdot \sum_{i=1}^n \tilde{x}_i - \tilde{X}_1.$$

- *Case study:* $n = 4$ cells,

- with the high spatial resolution accuracy $\sigma_h = 0.5$
- and the high spatial resolution estimates (in each of these cells)

$$\tilde{x}_1 = 2.0, \quad \tilde{x}_2 = 3.0, \quad \tilde{x}_3 = 5.0, \quad \tilde{x}_4 = 6.0;$$

- the corresponding low spatial resolution estimate is $\tilde{X}_1 = 3.7$.

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19. Estimates of High and Low Spatial Resolution: Illustration

$\tilde{x}_1 = 2.0$	$\tilde{x}_2 = 3.0$
$\tilde{x}_3 = 5.0$	$\tilde{x}_4 = 6.0$

$$\tilde{X}_1 = 3.7$$

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20. Numerical Example: Discussion

- We assume that the low spatial resolution estimate is accurate ($\sigma_l \approx 0$).
- So, the average of the four cell values is equal to the result $\tilde{X}_1 = 3.7$ of this estimate:

$$\frac{x_1 + x_2 + x_3 + x_4}{4} \approx 3.7.$$

- For the high spatial resolution estimates \tilde{x}_i , the average is slightly different:

$$\frac{\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3 + \tilde{x}_4}{4} = \frac{2.0 + 3.0 + 5.0 + 6.0}{4} = 4.0 \neq 3.7.$$

- *Reason:* high spatial resolution estimates are much less accurate: $\sigma_h = 0.5$.
- We use the low spatial resolution estimate to “correct” the high spatial resolution estimate.

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21. Numerical Example: Results

- Here, the correcting term takes the form

$$\lambda = \frac{\tilde{x}_1 + \dots + \tilde{x}_n}{n} - \tilde{X}_1 =$$
$$\frac{2.0 + 3.0 + 5.0 + 6.0}{4} - 3.7 = 4.0 - 3.7 = 0.3.$$

- So, the corrected (“fused”) values x_i take the form:

$$x_1 = \tilde{x}_1 - \lambda = 2.0 - 0.3 = 1.7; \quad x_2 = \tilde{x}_2 - \lambda = 3.0 - 0.3 = 2.7;$$

$$x_3 = \tilde{x}_3 - \lambda = 5.0 - 0.3 = 4.7; \quad x_4 = \tilde{x}_4 - \lambda = 6.0 - 0.3 = 5.7.$$

- For these corrected values, the arithmetic average is equal to the low spatial resolution estimate:

$$\frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{1.7 + 2.7 + 4.7 + 5.7}{4} = 3.7.$$

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22. The Result of Model Fusion: Simplified Setting

$\tilde{x}_1 = 1.7$	$\tilde{x}_2 = 2.7$
$\tilde{x}_3 = 4.7$	$\tilde{x}_4 = 5.7$

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23. Taking $\sigma_{e,j}$ Into Account

- *Idea:* take into account the requirement that
 - the actual values in each cell are approximately equal to \tilde{X}_1 ,
 - with the accuracy $\sigma_{e,1}$ equal to the empirical standard deviation.

- *Resulting formulas:* $\mu = \frac{\lambda}{\frac{1}{n} \cdot \sigma_h^2} = \frac{\frac{1}{n} \cdot \sum_{i=1}^n \tilde{x}_i - \tilde{X}_1}{\frac{1}{n} \cdot \sigma_h^2}$, and

$$x_i = \frac{\tilde{x}_i - \lambda}{1 + \frac{\sigma_h^2}{\sigma_{e,1}^2}} + \tilde{X}_1 \cdot \frac{\frac{\sigma_h^2}{\sigma_{e,1}^2}}{1 + \frac{\sigma_h^2}{\sigma_{e,1}^2}}.$$

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24. Taking $\sigma_{e,j}$ Into Account: Numerical Example

- *General idea:* the actual values in each cell are approximately equal to \tilde{X}_1 .
- *In our example:* $x_i \approx \tilde{X}_1$, with the accuracy

$$\sigma_{e,1}^2 = \frac{1}{4} \cdot \sum_{i=1}^4 (\tilde{x}_i - E_1)^2, \text{ where } E_1 = \frac{1}{4} \cdot \sum_{i=1}^4 \tilde{x}_i.$$

- Here, $E_1 = \frac{1}{4} \cdot \sum_{i=1}^4 \tilde{x}_i = \frac{\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3 + \tilde{x}_4}{4} = 4.0$, thus,

$$\sigma_{e,1}^2 = \frac{(2.0 - 4.0)^2 + (3.0 - 4.0)^2 + (5.0 - 4.0)^2 + (6.0 - 4.0)^2}{4} = \frac{4 + 1 + 1 + 4}{4} = \frac{10}{4} = 2.5.$$

- Hence $\sigma_{e,1} \approx 1.58$.

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25. Taking $\sigma_{e,j}$ Into Account (cont-d)

• *Reminder:* $x_i = \frac{1}{1 + \frac{\sigma_h^2}{\sigma_{e,1}^2}} \cdot (\tilde{x}_i - \lambda) + \frac{\frac{\sigma_h^2}{\sigma_{e,1}^2}}{1 + \frac{\sigma_h^2}{\sigma_{e,1}^2}} \cdot \tilde{X}_1.$

• Here, $\sigma_h = 0.5$, $\sigma_{e,1} = 2.5$, $\frac{\sigma_h^2}{\sigma_{e,1}^2} = \frac{0.25}{2.5} = 0.1$, so

$$\frac{1}{1 + \frac{\sigma_h^2}{\sigma_{e,1}^2}} = \frac{1}{1.1} \approx 0.91, \text{ and } \frac{\frac{\sigma_h^2}{\sigma_{e,1}^2}}{1 + \frac{\sigma_h^2}{\sigma_{e,1}^2}} \cdot \tilde{X}_1 = \frac{0.1}{1.1} \cdot 3.7 \approx 0.34;$$

$$x_1 \approx 0.91 \cdot (2.0 - 0.3) + 0.34 \approx 1.89;$$

$$x_2 \approx 0.91 \cdot (3.0 - 0.3) + 0.34 \approx 2.79;$$

$$x_3 \approx 0.91 \cdot (5.0 - 0.3) + 0.34 \approx 4.62;$$

$$x_4 \approx 0.91 \cdot (6.0 - 0.3) + 0.34 \approx 5.53.$$

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26. The Result of Model Fusion: General Setting

$\tilde{x}_1 \approx 1.89$	$\tilde{x}_2 \approx 2.79$
$\tilde{x}_3 \approx 4.62$	$\tilde{x}_4 \approx 5.53$

- The arithmetic average of these four values is equal to

$$\frac{x_1 + x_2 + x_3 + x_4}{4} \approx \frac{1.89 + 2.79 + 4.62 + 5.53}{4} \approx 3.71.$$

- So, within our computation accuracy, it coincides with the low spatial resolution estimate $\tilde{X}_1 = 3.7$.

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27. Model Fusion: Case of Interval Uncertainty

- We take into account three different types of approximate equalities:

- Each high spatial resolution estimate \tilde{x}_i is approximately equal to the actual value x_i :

$$\tilde{x}_i - \Delta_{h,i} \leq x_i \leq \tilde{x}_i + \Delta_{h,i}.$$

- Each lower spatial resolution value \tilde{X}_j is \approx to the average of values of all the cells $x_{i(1,j)}, \dots, x_{i(k_j,j)}$:

$$\tilde{X}_j - \Delta_{l,j} \leq \sum_i w_{j,i} \cdot x_i \leq \tilde{X}_j + \Delta_{l,j}.$$

- Finally, we have prior bounds $\underline{x}_{pr,i}$ and $\bar{x}_{pr,i}$ on the values x_i , i.e., bounds for which

$$\underline{x}_{pr,i} \leq x_i \leq \bar{x}_{pr,i}.$$

- Our objective is to find, for each $k = 1, \dots, n$, the range $[\underline{x}_k, \bar{x}_k]$ of possible values of x_k .

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28. Case of Interval Uncertainty: Algorithm

- The estimates lead to a system of linear inequalities for the unknown values x_1, \dots, x_n .
- Thus, for each k , finding the endpoints \underline{x}_k and \bar{x}_k means optimizing the values x_k under linear constraints.
- This is a particular case of a general linear programming problem.
- So, we can use Linear Programming to find these bounds:
 - the lower bound \underline{x}_k can be obtained if we minimize x_k under the constraints

$$\tilde{x}_i - \Delta_h \leq x_i \leq \tilde{x}_i + \Delta_h, \quad i = 1, \dots, n;$$

$$\tilde{X}_j - \Delta_l \leq \sum_i w_{j,i} \cdot x_i \leq \tilde{X}_j + \Delta_l; \quad \underline{x}_{pr,i} \leq x_i \leq \bar{x}_{pr,i}.$$

- the upper bound \bar{x}_k can be obtained if we maximize x_k under the same constraints.

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29. Conclusions

- We propose a fast practical alternative to joint inversion of multiple datasets.
- Specifically, we consider models that have
 - not only different accuracy and coverage,
 - but also different spatial resolution.
- To fuse such models, we must account for three different types of approximate equalities:
 - each high spatial resolution estimate is \approx the actual value in the corresponding cell;
 - each lower spatial resolution estimate is \approx to the weighted average of the cell values;
 - each lower spatial resolution value is \approx the value within each of the constituent cells.

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30. Conclusions (cont-d)

- *Possible situations*: probabilistic or interval uncertainty.
- *Solution*: use the least squares or interval technique to combine the approximate equalities.
- *Example*: the least squares approach
 - we find the desired combined values
 - by minimizing the resulting sum of weighted squared differences.
- *Case study*: simulated (synthetic) geophysical data.
- *We show*: that model fusion indeed improves the accuracy and resolution of individual models.
- *Future plans*: apply model fusion techniques to more realistic simulated data and to real geophysical data.

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