A general purpose software for reliability-based optimal design

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Abstract. Due to the inherent uncertainties in loads, materials and manufacturing quality, variabilities are unavoidable in structural responses and can cause significant deviations in expected product performance. To ensure the reliability of a structure, these uncertainties must be considered during structural design.
In cases where a structure is optimized such that the design requirements are barely met, it is important for engineers to asses the design's reliability and, if necessary, optimize the structure considering the reliability requirement. In those situations where variations in design and operating environment result in considerable variability of the performance of a system, it is critical for engineers to search for a robust design. A robust design is one in which the performance deviation is within acceptable limits.
The objective of this paper is to present a life cycle management tool implemented within a general purpose stochastic analysis software named COSSAN-X$^{\text{TM}}$. An overview of the tools for reliability and optimization implemented in the code which are required to solve the robust design problem is presented. In addition, the interaction with deterministic Finite Element codes is also addressed. The salient features of the software are presented by means of a challenging problem of Reliability based optimization and life cycle management.

Keywords: Stochastic Analysis, Software, Robust Optimization, Reliability, Reliability-based optimization, Maintenance Scheduling, Fatigue Crack Propagation

1. Introduction

Within the community of computational mechanics, currently only deterministic structural analyses are performed; nonetheless, such deterministic analyses provide insufficient information to capture the variability of the structural response due to the inevitable uncertainties in loads, materials and manufacturing quality. Although nowadays quite detailed and refined models can be handled, the accuracy in predicting actual responses has not improved in the same manner.
Structural models might contain thousands of input parameters which are not known precisely. However, when one uses the available general purpose Finite Element (FE)-codes, these parameters are generally set to deterministic values. Hence, the deterministic structural analysis provides only a single result out of an infinite set of possible solutions. Therefore, in order to obtain reliable results, deterministic analysis should be repeated for all the possible values of the uncertain input parameters adopting different stochastic analysis methods (e.g. Monte Carlo simulation, Stochastic Finite Element Analysis, etc.).

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Recently, the concept of robust design has gained much attention, see e.g. (Beyer and Sendhoff, 2007). Robust design is a proven development philosophy focused on optimal design solutions considering uncertainties. Approaching this challenging goal requires that robust design principles constitute an early and integral part of the development cycle. The objective is to make the end-product, i.e. the structure, less sensitive to factors that could adversely affect performance. Among the different specific approaches for robust design, Reliability-based Optimization (RBO) is a salient methodology. RBO consists in solving an optimization problem while explicitly considering the effects of uncertainty; these effects are accounted for by means of probabilities of occurrence and expected values. The field of RBO is wide and one of its most important applications is on the solution of life cycle management problems, e.g. scheduling of maintenance activities in structural systems prone to accumulate damage.
Although stochastic and robust design methods offer a much more realistic approach for analysis and design of structural systems, their application has remained limited. One of the reasons to explain this fact is that the development of software for stochastic analysis has received considerable less attention than its deterministic counterpart. However, a recent survey (Pellissetti and Schuëller, 2006) indicates that this trend may change radically in the near future. Within this context, the purpose of this paper is presenting a general purpose software, named COSSAN-X, developed at the Institute of Engineering Mechanics of the University of Innsbruck, Austria, EU. Here, the main features and the innovative aspects of the software for solving problems of robust design are presented. Finally, the application of the software is exemplified by solving a challenging problem of RBO and life cycle management.
The outline of the present paper is the following. In the next Section, a brief overview on robust design, RBO and their application for solving life cycle management problems is addressed. In Section 3, an overview of the components and capabilities with respect to robust design of the software COSSAN-X is presented. Section 4 demonstrates the capabilities of COSSAN-X for solving a maintenance scheduling problem of a fatigue-prone metallic component. The contribution is closed with some final remarks in Section 5.

2. Theoretical Background

2.1. Robust design and Reliability-based Optimization

Optimization procedures offer a sound basis for design in engineering (Arora (Ed.), 2007; Haftka and Gürdal, 1992), as they allow identifying a set of *design variables* that fulfills best a certain *objective* while enforcing a number of *constraints*. That is, optimization provides the means for *selecting features* associated with a particular system (such as dimensions of structural elements, inspection and repair intervals, etc.) that maximizes (or minimizes) an appropriate *target function*, such as operation costs, benefits, etc.; in addition, a number of *performance objectives* – such as serviceability conditions – should be satisfied at the final configuration.
In realistic situations, most of the parameters of the system that affect the performance are of an uncertain nature; typical examples of these parameters refer to loadings, geometrical properties, material properties, temperature of operation, damage accumulation processes, etc. Due to these

uncertainties, the performance of the system could be subjected to a large variability, rendering an optimal design – determined assuming deterministic parameters – into an unfeasible solution. In order to avoid such undesirable design solutions, the optimal solution should be chosen considering its associated degree of robustness. In this context, a robust design is a configuration of the system that is relatively invariant with respect to parameter changes, production tolerance, model sensitivities and other uncertain conditions. The process of finding such solution is referred to as robust design optimization (Mulvey et al., 1995; Kalsi et al., 2001; Park et al., 2006; Beyer and Sendhoff, 2007).

Early efforts for considering the effect of uncertainties in design processes (within the context of manufacturing) are closely connected with the *Taguchi method* of robust design (Taguchi, 1989). In this method, the objective is selecting a set of control factors that optimizes a certain loss function that is affected by noise factors (i.e. uncertainties). In the solution of the optimization problem, design of experiments is used instead of optimization.

The Taguchi method is certainly not the only alternative for robust design. In fact, different robustness measures can be defined in order to account for the effects of uncertainties. For example, probability theory offers appropriate tools for addressing uncertainties rationally (Freudenthal, 1956) by characterizing the uncertain parameters as random variables. In this way, the effects of uncertainty can be quantified in terms of *structural reliability* (or its complement, the probability of failure) and of *expected costs*, i.e. the costs associated with the occurrence of a particular event within the life time of a system. The incorporation of these metrics in the optimization problem leads to a particular class of robust design problems, namely Reliability-based Optimization (RBO) problems, see e.g. (Ching and Hsieh, 2007; Enevoldsen and Sørensen, 1994; Gasser and Schuëller, 1997; Jensen and Catalan, 2007; Kupfer and Freudenthal, 1977; Marti (Ed.), 1997; Papadrakakis et al., 2005; Taflanidis and Beck, 2008b). The scope and solution of RBO problems are discussed in more detail below.

2.2. Reliability-based optimization: Formulation

RBO constitutes a powerful tool for design in engineering, as it offers a systematic approach for taking decisions under uncertainty. In fact, the RBO methodology can be applied to solve a wide spectrum of problems (Moses and Kinser, 1967; Enevoldsen and Sørensen, 1994). In particular, a problem of much relevance in engineering is the minimization of expected life time costs associated with a certain system considering maintenance costs and eventual failure; this problem is also known as a *life cycle management problem.* Its formulation considers costs due to partial damage and structural collapse (Kupfer and Freudenthal, 1977). In mathematical terms, this problem is defined as shown below:

$$E\left[C(\boldsymbol{y},\boldsymbol{\theta})\right], \quad \boldsymbol{y} \in \Omega_y \tag{1}$$

subject to:

$$h_i(\boldsymbol{y}) \leq 0, \quad i = 1, \ldots, n_C \tag{2}$$

$$p_j(\boldsymbol{y}) \leq p_j^{tol}, \quad j = 1, \ldots, n_P \tag{3}$$

In the Equations above, $\boldsymbol{y}$ denotes the vector of design variables, which are those variables that can be selected among a certain set and that influence the performance of a structural system or trigger specific events; $\boldsymbol{\theta}$ denotes the vector uncertain parameters, which are characterized by means of a joint probability density function $f(\boldsymbol{\theta})$; h_j are constraints of the problem (e.g. side constraints on $\boldsymbol{y}$); C is a cost function (which can eventually be a random variable depending on $\boldsymbol{\theta}$) and $E[\cdot]$ is the expectation operator; finally, p_j denotes the probability of occurrence of the j-th event, which should be equal or smaller than a certain tolerable threshold p_j^{tol}. The terms $E\left[C(\boldsymbol{y},\boldsymbol{\theta})\right]$ and $p_j(\boldsymbol{y})$ in Equations (1) and (3), respectively, are defined by means of the multi-dimensional integrals shown below:

$$E\left[C(\boldsymbol{y},\boldsymbol{\theta})\right] = \int_{g(\boldsymbol{y},\boldsymbol{\theta})\leq 0} C(\boldsymbol{y},\boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{4}$$

$$p_j(\boldsymbol{y}) = P\left[g_j(\boldsymbol{y},\boldsymbol{\theta}) \leq 0\right] = \int_{g_j(\boldsymbol{y},\boldsymbol{\theta})\leq 0} f(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{5}$$

In Eqs. (4) and (5), $g(\boldsymbol{y},\boldsymbol{\theta})$ is the so-called *performance function*, which is a function used to model the performance objectives associated with a specific system. It is defined such that $g(\boldsymbol{y}^\star,\boldsymbol{\theta}^\star)$ is smaller or equal to zero when the pair $(\boldsymbol{y}^\star,\boldsymbol{\theta}^\star)$ causes an unacceptable performance of the system; otherwise, $g(\boldsymbol{y}^\star,\boldsymbol{\theta}^\star) > 0$. Usually, the evaluation of the performance function for a specific realization of the design variables and random parameters requires the computation of the response of a system by means of a simulation model (such as a FE model).

The application of the RBO methodology to problems of engineering interest can be quite challenging due to high numerical costs involved in its solution. This is due to the fact that the structural response must be evaluated by means of virtual simulation models several times for different sets of design variables and uncertain parameters. In other words, the RBO problem is a *double-loop problem* (Enevoldsen and Sørensen, 1994), i.e. the reliability evaluation algorithm is nested within the optimization loop. The numerical costs associated with such formulation are usually unaffordable (except by the case of academic examples). Therefore, methods for solving RBO problems seek the introduction of simplifications or special formulations for reducing the numerical efforts. For example, the development of approximate reliability methods (see, e.g. (Breitung, 1994)) and advanced simulation methods (see, e.g. (Au and Beck, 2001; Katafygiotis et al., 2007; Schuëller et al., 2005)) allow the estimation of probabilities of failure and expected costs most efficiently. Furthermore, the application of meta-modeling techniques has allowed replacing numerically intensive simulation models by inexpensive ones (see, e.g. (Hurtado, 2004; Pichler et al., 2009; Zhang and Foschi, 2004)). The introduction of efficient strategies and approximation concepts also play a fundamental role in yielding challenging RBO problems tractable (Ching and Hsieh, 2007; Jensen et al., 2009; Gasser and Schuëller, 1997; Taflanidis and Beck, 2008a). In addition, the advent of High Performance Computing (HPC) and the parallelization of the algorithms allow the possibility of performing demanding numerical simulations in reduced time, see e.g. (Pellissetti, 2009; Thierauf and Cai, 1997; Umesha et al., 2005).

2.3. Application of Reliability-based Optimization for Maintenance Scheduling

One of the most relevant applications of RBO is life cycle management and – in particular – the scheduling of maintenance activities for fatigue-prone metallic components. This is due to the

fact that the development of fatigue cracks in metallic structures operating under cyclic loading is a highly uncertain phenomenon (Koutsourelakis et al., 2006; Virkler et al., 1977). Although maintenance activities can be a cost-effective measure for coping with damage accumulation (Rocha and Schuëller, 2005; Shiao, 2005), their scheduling is quite challenging due to the uncertainties in the crack propagation. In this involved decision-making scenario, RBO arises as a natural choice for scheduling maintenance activities (Gasser and Schuëller, 1997; Kupfer and Freudenthal, 1977; Madsen et al., 1991; Skjong, 1985; Thoft-Christensen and Sørensen, 1987; Valdebenito and Schuëller, 2009; Yang, 1993).
During the life time T of a particular mechanical component that develops, n_C, fatigue cracks, several different events may take place (see, e.g. (Crémona and Lukić, 1998; Faber et al., 1996; Madsen et al., 1991)). That is, the mechanical component may survive or collapse; at the inspection time ($T_I,\ 0 < T_I < T$), a crack may be detected by the inspection procedure or it may be missed; finally, upon detection of the crack, a repair action may or may not take place. The occurrence of any of these events will be defined by the physics of the problem (in this case, the crack propagation phenomenon), the maintenance schedule and the uncertain parameters (which are denoted as $\boldsymbol{\theta}$). Thus, the problem of optimal design of a maintenance schedule can be interpreted as the identification of a set of design variables $\boldsymbol{y}$ (such as quality and frequency of inspection, technique for repairing, etc.) that minimizes the total cost, C_T, associated with the operation of a mechanical component during its life time. In this context, the total cost function is modeled as the summation of the cost associated with inspection activities, C_I, repair, C_R, and eventual collapse, C_F, (Enevoldsen and Sørensen, 1994; Kupfer and Freudenthal, 1977). The cost function is a random variable itself, as it depends on the occurrence of the aforementioned repair and failure events which – in turn – depend on $\boldsymbol{\theta}$. Then, the problem of minimization of C_T can be addressed by defining an appropriate deterministic substitute problem, e.g. by taking the expected value of the total cost function (Marti and Stoeckl, 2004). Thus, the RBO problem associated with maintenance scheduling is defined in mathematical terms as:

$$\min \quad E\left[C_T(\boldsymbol{y},\boldsymbol{\theta})\right] = E\left[C_I(\boldsymbol{y},\boldsymbol{\theta})\right] + E\left[C_R(\boldsymbol{y},\boldsymbol{\theta})\right] + E\left[C_F(\boldsymbol{y},\boldsymbol{\theta})\right], \quad \boldsymbol{y} \in \Omega_y \tag{6}$$

subject to:

$$h_j(\boldsymbol{y}) \leq 0, \quad j = 1, \ldots, n_{DC} \tag{7}$$

The expected costs associated with inspection, repair and failure can be approximated as the product between the cost associated with each of the aforementioned events and the probability of occurrence of the event (Gasser and Schuëller, 1997), as shown below.

$$E[C_x(\boldsymbol{y},\boldsymbol{\theta})] \approx c_x p_x(\boldsymbol{y}) \tag{8}$$

$$E[C_x(\boldsymbol{y},\boldsymbol{\theta})] \approx c_x \int_{g_x(\boldsymbol{y},\boldsymbol{\theta})} f(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{9}$$

In Eqs. (8) and (9), the subscript $(\cdot)_x$ indicates a quantity associated with the event x, i.e. inspection (I), repair (R) or failure (F); $p_x(\cdot)$ measures the probability of occurrence of the event x. For calculating the expected cost associated with inspection, a common assumption is setting $p_I(\cdot)$ equal to one, as the probability that a mechanical component fails before inspection is quite low. For calculating the expected costs associated with repair and failure, it is necessary to evaluate the probability of occurrence of each of the events $p_R(\cdot)$ and $p_F(\cdot)$, respectively. The definition of the associated performance functions ($g_R(\cdot,\cdot)$ and $g_F(\cdot,\cdot)$, respectively) is based on the physics of the problem at hand. For example, in case the crack propagation phenomenon is modeled within the context of Elastic Fracture Mechanics (see, e.g. (Anderson, 1991; Kanninen and Popelar, 1985)) considering mode I loading, the performance function associated with the failure event is defined as:

$$g_F(\boldsymbol{y},\boldsymbol{\theta}) = K_{Ic}(\boldsymbol{y},\boldsymbol{\theta}) - K_I(\boldsymbol{y},\boldsymbol{\theta}) \tag{10}$$

where $K_{Ic}(\cdot,\cdot)$ is the fracture toughness associated with the mechanical component and $K_I(\cdot,\cdot)$ is the stress intensity factor associated with the crack present in the mechanical component.
It is important to note that the evaluation of the probability integral in Eq. (9) can be quite challenging, as the dimension of the vector of uncertain parameters $\boldsymbol{\theta}$ can be quite large and also due to non linearities of the performance function $g_x(\cdot,\cdot)$. Thus, simulation methods – in particular, the so-called advanced simulation methods – are the best alternative for evaluating this integral accurately and efficiently (Schuëller and Pradlwarter, 2007).

2.4. Solution Strategy

Several approaches have been proposed in order to solve the RBO problem posed in Eq. (6), such as approximation concepts, construction of meta-models, stochastic search methods, etc. For an overview on these methods, it is referred to (Schuëller and Jensen, 2008). In the particular case of this contribution, a decoupling approach is applied in order to solve the RBO problem most efficiently.
A decoupling approach is a very simple yet effective strategy. It consists in the separation of optimization and reliability analysis. The key issue for applying a decoupling approach is the construction of an approximate representation of the probabilities. Early efforts demonstrated that a probability function may be approximately represented by means of a response surface (Murthy and Subramanian, 1968; Lind, 1976; Kanda and Ellingwood, 1991). This approach was employed in the context of RBO in (Gasser and Schuëller, 1997), in order to generate a *global* approximation of the failure probabilities as an explicit function of the design variables; in this context, *global* refers to the fact that the approximation is assumed to be valid over the whole domain of the design variables. The approximation is constructed by selecting some predefined interpolation points in the space of the design variables, at which the failure probability is calculated by means of simulation; then, a response surface is adjusted to the data collected at the interpolation points in a least square sense. The approach introduced in (Gasser and Schuëller, 1997) was further extended in (Jensen and Catalan, 2007; Jensen, 2005), where it was shown that the construction of *local* approximations of the failure probabilities could be much less involved. The approach using local approximations can be incorporated in a sequential approximate optimization framework, see e.g. (Haftka and Gürdal, 1992), in order to solve the target RBO problem.

3. COSSAN-X

3.1. General remarks

As shown in Section 2, robust design constitutes a very powerful tool for design in engineering, as it possesses several advantages over its deterministic counterpart. However, it is also noted that for its implementation requires a number of specific numerical procedures. Moreover, most of these procedures are not implemented, usually, in finite elements codes commonly used in engineering practice.

In view of this issue, this Section discusses a general purpose stochastic analysis code intended for a wide range of applications within the field of stochastic structural mechanics which is developed at the Institute of Engineering Mechanics, University of Innsbruck, Austria, EU under the collective name COSSAN™ (COmputational Stochastic Structural ANalysis) (Schuëller and Pradlwarter, 2006). The latest version, namely COSSAN-X, is coded in a object oriented fashion in MATLAB® environment, which provides an expandable modular framework. The program layers of COSSAN-X are represented in the Figure 1.

COSSAN-X provides different interfaces: a general purpose graphical user interface, a MATLAB command line interface especially geared for researchers that want to try and implement new algorithms and new solution sequences, and finally, dedicated plug-ins for commercial software (e.g. Patran®).

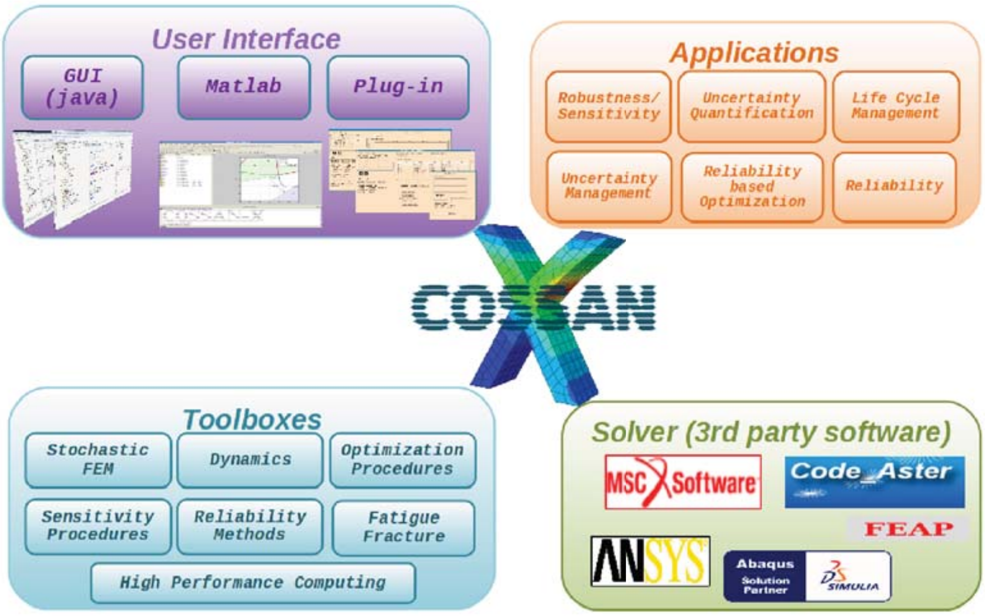

Figure 1. Schematic representation of the general purpose software COSSAN-X.

COSSAN-X provides a series of different communication tools to interact with standard FE packages, i.e. solver, used in industry, such as Nastran, Ansys, Abaqus, code_Aster etc. The software modularity allows adding easily new communication tools with other third-party software.

The Toolboxes-layer represents the core component of the software and implements the state of the art in stochastic structural analysis that have been shown to represent a robust and efficient approach for the stochastic analysis, see e.g. (Schuëller and Pradlwarter, 2009; Schuëller, 2009).

The combination of these algorithms into specific solution sequences permits solutions of engineering problems, which form the Applications-layer, such as robust reliability optimization analysis, life cycle management, etc.

3.2. The role of general purpose software

The term *general purpose software* means that a reasonably wide array of engineering and scientific problems can be tackled by the same software. This is in contrast with specialized software that is developed for solving only a specific type of problems of a particular discipline.

In general, the capabilities of the specialized software are not completely covered by the general purpose software, as they can only solve the problem for which they have been designed for. On the other hand, general purpose software is much more flexible and the advantages for the user are manifold, especially the advantage to have only one software that is able to solve different problems. This involves a drastic reduction of the amount of time that an analyst needs to invest in learning how to use the software.

Furthermore, general purpose software offers the possibility to customize the solution sequence becoming able to solve problems that were not even prefigured during the design of the software.

The downside of a general purpose software is that, usually, it is much more complex than dedicated software in terms of number of lines of code required, structure and the time required to be developed and tested. However, since a general purpose software is designed to solve a broad variety of problems, it is also much simpler to use.

With respect to reliability-based optimal design, a general purpose software should include a variety of solution algorithms from which the user can choose to perform the analysis. Since all the algorithms can be adequately applied only over a certain limited range of problems, a greater selection will automatically increase the number of engineering applications that can be efficiently tackled. Many existing software packages provide an implementation of a particular method or approach rather than offering the entire array of options.

COSSAN-X provides a large set of different tools and methods that can be combined to solve a specific problem as shown in the Section 3.4.

3.3. Interaction with 3rd party software

Nowadays the structural analysis is performed almost exclusively by FE-packages. These packages are continuously improved and updated to account for new trends and developments in structural analysis. In general, an analyst is able to solve a deterministic problem using a specific solver, for instance a commercial FE-solver or an in-house solver. Of course, the analyst aims to solve the stochastic problem using the same FE-models that they are already familiar with. However, these FE-packages are generally not designed to account for any uncertainty in the structural parameters or in the load conditions.

COSSAN-X provides a series of different communication tools to interact with standard FE packages, i.e. solver, used in industry, such as Nastran, Ansys, Abaqus, code_Aster etc. The interaction is based on the manipulation of the ASCII input/output files; this strategy allows to include theoretically any type of 3rd party software which uses input/output files in a ASCII format. The software modularity allows adding easily new communication tools with other third-party software.

The basic concept is to use the original ASCII input files of the 3rd party software as a basis and to modify these files for generating automatically valid input files for the stochastic analysis. For this purpose the quantities associated with uncertainty that are being analyzed are replaced in the original ASCII input files by some identifiers, i.e. strings of unique characters.

COSSAN-X provides post-processor tools able to extract the quantity of interest from the output files of the 3rd party software. Starting from the original output files, i.e. the output files resulting from the deterministic analysis, the relative and/or absolute positions of the quantities of interest are defined and successively used to extract the values from the output files.

These identifiers establish the relation between the variables defined in COSSAN-X and the physical variables defined in the 3rd party software. The process of injecting the parameters in the input files and the extraction of the quantity of interest from the output files is totally automatized.

3.4. Life cycle management

As shown in Figure 1, the toolboxes-layer represents the core component of the software. The combination of the Reliability toolbox, Optimization toolbox and High Performance Computing toolbox allows solutions, for instance, of robust reliability optimization analysis and life cycle management as shown in Figure 2.

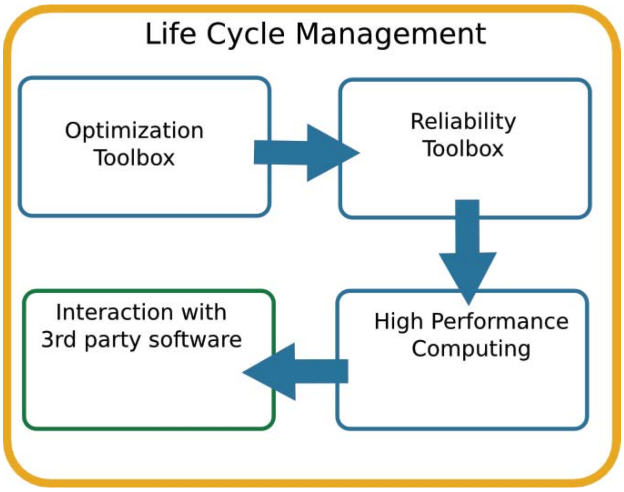

Figure 2. COSSAN-X: Life Cycle Management application

3.4.1. *Reliability toolbox*

The objective of the reliability toolbox is to quantify the failure probability of the structure under investigation. The failure probability involves the calculation of the multidimensional integral defined in Eq. (5).

Mechanical systems arising in typical engineering applications usually exhibit a degree of complexity that prevents an analytical solution of the failure probability since $g(\boldsymbol{y}, \boldsymbol{\theta})$ is not known explicitly. The failure probability can always be estimated via Monte Carlo simulation. However, for most practical applications the evaluation of a single point of the function $g(\boldsymbol{y}, \boldsymbol{\theta})$, typically, requires a FE-analysis of the structural model and the computational time required by the Monte Carlo simulation becomes unfeasible.

The failure probability is estimated in the reliability toolbox of COSSAN-X by means of approximate methods (see e.g. (Ditlevsen and Madsen, 1996)) or advanced simulation-based methods (see e.g. (Schuëller et al., 2004)). To the first category the following methods belong to: First Order Reliability Method (FORM), Second Order Reliability Method (SORM) and bounds. The family of the advanced simulation-based methods includes Importance Sampling, Line sampling and Subset sampling.

Furthermore, COSSAN-X can estimate the probability of occurrence of a single failure model of a structure as well as the probability of occurrence of certain combination of events, the so-called cut-sets of the reliability assessment for structures with multiple failure modes, i.e. system reliability analysis.

3.4.2. *Optimization toolbox*

The optimization toolbox of COSSAN-X provides a set of widely used algorithms for standard and large-scale optimization. The algorithms implemented can be adopted to solve constrained and unconstrained continuous and discrete problems.

The field of optimization is vast, existing several ways to classify the available algorithms (see e.g. (Arora (Ed.), 2007; Spall, 2003; Bonnans et al., 2003)). Among these possibilities, a classical criterion is distinguishing between gradient-based algorithms and the gradient-free algorithms (the latter are also known as zeroth-order or direct optimization methods). The reason for selecting this classification is due to the fact that the computation of gradients when using a gradient-based algorithm may be rather involved, especially when the functions are noisy. However, gradient-free algorithms usually require much more function evaluations than gradient-based algorithms to solve the same problem, although they are better to handle noisy functions; moreover, some of the gradient-free algorithms are also capable of performing global optimization. Therefore, the classification of gradient-based and gradient-free optimization algorithms reflects the trade-off that exists between these two types of methods.

The COSSAN-X optimization toolbox includes the gradient-based optimization method Sequential Quadratic Programming and the following gradient-free algorithms: Genetic Algorithms, Simulated Annealing, Constrained Optimization by Linear Approximation (COBYLA), Cross Entropy and Evolution Strategies.

3.4.3. *High Performance Computing toolbox*

A common limitation of all of the stochastic analysis methods is that the computational cost of performing an analysis is a multiple (often by orders of magnitude) of the computational cost of a deterministic analysis performed with a given numerical (finite element) model. This is because instead of running the analysis code only once, the stochastic analysis involves its repeated execution. Performing uncertainty analysis might require large computational cost especially for detailed models.

COSSAN-X offers the possibility to perform stochastic analysis adopting the immense computational power and the great opportunity provides by the grid computing (e.g. (Pellissetti, 2009; Pellissetti and Schuëller, 2009)). In fact, grid computing offers - at relatively low costs - more flexibility than the traditional parallel execution of the code in very expensive supercomputers (Magoules et al., 2009). Furthermore, interfacing with job-managers (e.g. Sun Grid Engine (Sundaram et al., 2006)) COSSAN-X allows to distribute the execution of the solver on the available (remote) resources on a computer grid and maximizes the use of the available licenses while reducing the execution time (wall clock time) of the analysis task. Moreover, the job-managers allow to perform the analysis on a heterogeneous network, i.e. a grid composed by machines running different operating system. Thanks to the high performance computing toolbox it becomes possible, for instance, running COSSAN-X on a Windows or MAC OS platform and the 3rd party software, i.e. the FE-model, running on a Linux machine.

4. Numerical example

4.1. General Remarks

For the numerical example, the design of an optimal maintenance schedule for a fatigue-prone structural component is considered. In particular, the objective is selecting an inspection time T_I such that the expected total cost of operation is minimized, i.e. the expected cost of inspection, repair and failure. The component shown in Figure 3 consists of a symmetric plate including four identical rivet holes under axial tension; this plate models a small part of the fuselage of an aircraft. It should be noted that several fatigue cracks may develop at sites of stress concentration (around the holes), causing a drastic reduction of the life time.

4.2. Description of the problem

The load over the element is such that the range of the far-field stress is $\Delta\sigma = 90\ MPa$ and the far-field maximum stress, $\sigma_{\max} = 100\ MPa$; during one year of operation, a total of 1.2×10^5 load cycles are applied. The plate must endure a life period of 10 years. Therefore, the inspection time should be such that $0 < T_I < 10$ year.

It is assumed that at the beginning of this period, the element has already developed fatigue cracks; more specifically, there are a total of 8 cracks emanating from the rivet holes. The uncertainty in the length of these initial cracks ($a_{0,i},\ i = 1, \ldots, 8$) is characterized by means of a log-normal distribution with expected value 1 mm and standard deviation 0.5 mm, i.e. $a_{0,i} \sim \mathrm{LN}(1, 0.5)$. For the sake of simplicity, the random variables modeling the initial crack length are assumed to be uncorrelated; however, it has been observed that there is a considerable correlation between the length of cracks emanating from the same hole (Shoji et al., 2001).

The uncertainties in the inspection activities are accounted for by means of the probability of detection (POD), i.e. the probability that a crack of length a is detected by inspection. Different inspection techniques have associated different POD (Kuntiyawichai, 2005; Zheng and Ellingwood,

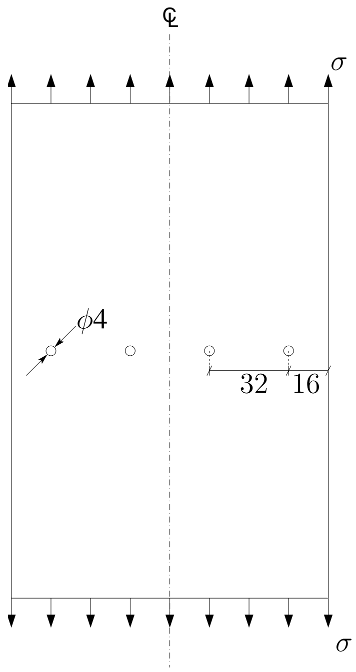

Figure 3. Schematic representation of plate with rivet holes (dimensions are in $[mm]$)

1998). In this example, the POD considered is given by the curve in Figure 4.

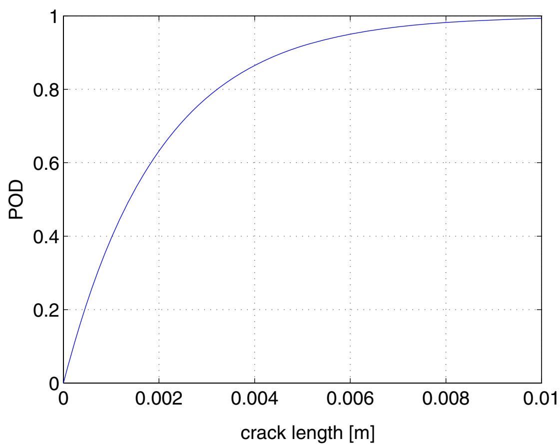

Figure 4. Probability of Detection (POD) as a function of the crack length

The crack propagation phenomenon (due to cyclic loadings) is treated within the framework of Elastic Fracture Mechanics. In particular, the crack propagation phenomenon is studied using the Paris-Erdogan law (Paris and Erdogan, 1963), which is a well-known model that has been thoroughly used in the literature. This law is expressed as follows.

$$\frac{da}{dN} = C(\Delta K)^m, \quad a(N=0) = a_0 \tag{11}$$

In Eq. (11), a is the crack length; N represents the number of load cycles; C and m are material properties; a_0 is the initial crack length; ΔK is the stress intensity factor (SIF) range, which characterizes the stress field at the tip of the crack. The parameters of the Paris-Erdogan law are modeled such that $m = 2$ and $C \sim \mathrm{LN}(2.5 \times 10^{-23}, 2.5 \times 10^{-24})$ $Pa^{-2}\text{cycles}^{-1}$. The stress intensity factors associated with the problem are calculated using the Finite Element Alternating Method (Wang and Atluri, 1996), which has been shown to be a feasible method for studying crack propagation problems considering uncertainties (Proppe, 2003; Proppe et al., 2002).

The failure event – the collapse of the mechanical component due to crack propagation – is modeled using the so-called R6 curve criterion (see, e.g. (Anderson, 1991; Kanninen and Popelar, 1985)). In this criterion, failure can occur due to a combined effect: interaction between brittle fracture (the SIF exceeding the fracture toughness, as described in Eq. (10)) and ductile failure (i.e. applied force over mechanical component exceeding its capacity). Concerning the parameters involved in the failure criterion, the fracture toughness is characterized by a log-normal distribution, i.e. $K_{Ic} \sim \mathrm{LN}(80, 8) MPa\ m^{-0.5}$. The yield stress is also characterized by a log-normal distribution with $\mu_{\sigma_Y} = 325\ MPa$ and coefficient of variation of 10%.

Finally, the costs associated with the events of inspection, repair and failure are set equal to $c_I = 50\ MU$ (where MU denotes *monetary units*), $c_R = 600\ MU$ and $c_F = 2{\times}10^5\ MU$, respectively.

4.3. Solution Strategy in COSSAN-X

The example addressed in this Section corresponds to a life cycle management problem. Its solution demands the estimation of probabilities of occurrence of the repair and failure events (cf. Eqs. (8) and (9)). In turn, the probability assessment requires the simulation of the crack propagation phenomenon. Finally, in order to find the optimal solution, it is necessary to evaluate the aforementioned probabilities for different values of the design variable (in this case, the time of inspection).

COSSAN-X offers a most appropriate platform for solving life cycle management problems. As shown in Section 3, COSSAN-X includes a number of tools for performing optimization, reliability assessment and communication with third party software. Here, the different toolboxes are organized as shown in Figure 2 in order to determine the optimal time of inspection. At the bottom level, the routine for simulating crack propagation (denoted as 3rd party software) is connected to the tool for assessing probabilities (reliability toolbox) using the high performance computing tool. In particular, Subset Simulation – which is an advanced simulation method developed in (Au and Beck, 2001) – is used for assessing the probabilities of repair and failure, respectively. As the high performance computing tool is applied, it is possible to simulate the crack propagation for different realizations of the vector of uncertain parameters in parallel. Finally, at the top level, the optimization toolbox is linked with the reliability toolbox in order to explore different maintenance strategies and determine the optimal one.

Table I. Speedup in training Response Surface due to the application of the HPC toolbox

Number of CPU's	Speedup factor
2	1.3
4	2.6
16	9.4
24	12.8
28	14.4
32	15.8

Although the solution strategy described in Figure 2 is feasible and can be implemented using COSSAN-X, it can be still numerically demanding – even when parallel computing is used – as the simulation of crack propagation is time-consuming. Therefore, a possible means for reducing the numerical efforts is training a *meta-model* of the crack propagation routine. Within COSSAN-X, the construction of a meta-model is straightforward, as a dedicated toolbox allows defining training and validation data for constructing the approximate model. In this example, a response surface of the routine for crack propagation is adopted. In order to speed-up the training phase of this meta-model, the HPC toolbox is applied in order to compute the response of the third party software for the different realizations of training and validation data. Again, the application of this toolbox is straightforward: once the calibration of the meta-model has been implemented sequentially (i.e. considering only one CPU), only a few additional commands are required to parallelize and distribute the calculations automatically on the available machines. Thus, the application of the HPC toolbox allows reducing the wall clock-time for training and validation phases of the aforementioned response surface, as indicated in Table I. In this Table, the speed-up indicates the factor in which the execution time is reduced w.r.t. the sequential computation.

The strategy for solving the RBO life cycle management problem described above is shown schematically in Figure 5.

Although the solution strategy proposed in Figure 5 is numerically more efficient than the approach proposed in Figure 2, it is possible to improve even further the efficiency by introducing a second meta-model, involving the expected total cost function (see Eqs. (6) and (8)) and the time of inspection. Thus, once this second meta-model is trained, the optimization problem can be solved with negligible numerical efforts. This solution strategy is implemented in this contribution and is illustrated schematically in Figure 6. It is important to mention that for solving the optimization problem, the gradient-free algorithm COBYLA (Bös, 2006; Powell, 1998), implemented in the optimization toolbox, has been applied, as it has been shown to be a very robust optimization technique.

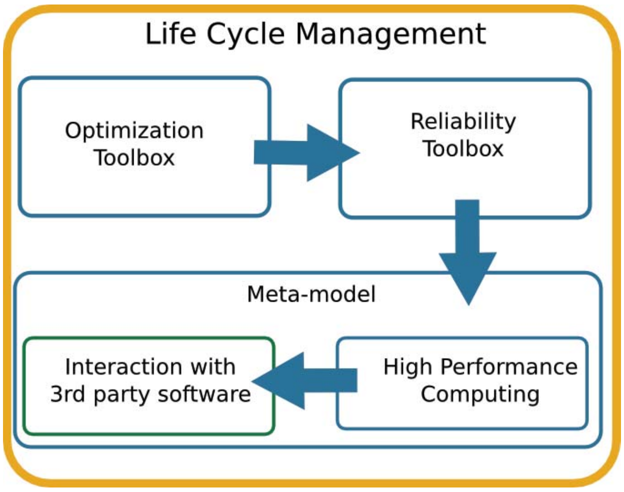

Figure 5. Solution strategy considering a meta-model for the crack propagation phenomenon.

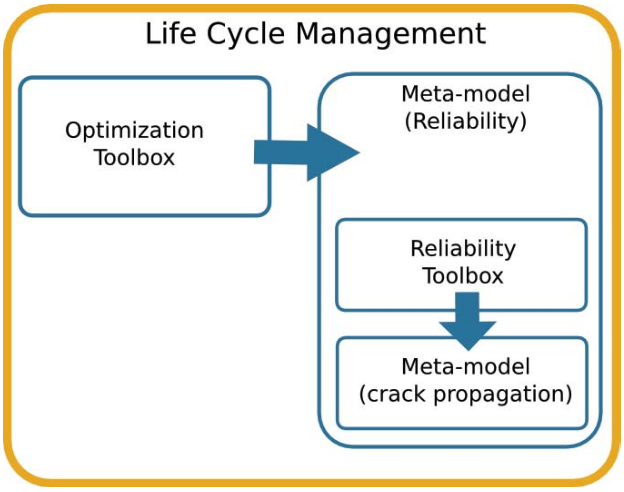

Figure 6. Solution strategy considering a meta-model for the reliability of the component shown in Figure 3.

4.4. Results

Prior to the design of the optimal maintenance strategy, the dependence of p_R and p_F on the time of inspection is analyzed. For this purpose, the values of these probabilities are estimated over a suitable grid, as shown in Figure 7. Concerning the repair activities, it is noted that p_R increases with the time of inspection, as the average crack size increases with time since that, generally, a larger crack has a higher probability of being detected (see also Figure 4). Regarding the failure event, it can be observed that p_F increases for both an early inspection time (because cracks are too small to be detected) and a late inspection time (because failure occurs before inspection). These observations are in agreement with other contributions in the literature (Grooteman, 2008; Yang and Trapp, 1974). In particular, the effectiveness of early inspection is quite limited, as cracks are too small and thus, difficult to detect; therefore, the beneficial effect of maintenance activities is lost.

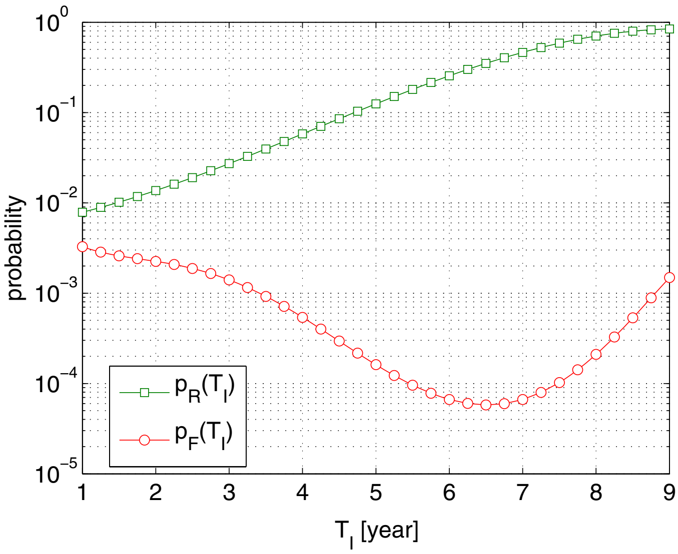

Figure 7. Probability of repair (p_R) and failure (p_F) as a function of the time of inspection (T_I).

In a next step, the RBO problem of optimal maintenance scheduling is solved using the approach described in Sections 2.4 and 4.3. Specifically, a response surface of the expected total cost function is generated by evaluating the cost function at five points. The response surface is modeled as a quadratic polynomial. Then, optimization is performed using this meta-model: the optimal time of inspection is found to be $T_I^{\text{opt}} = 4.8$ years and the associated value of the expected cost function is 155 MU. In order to gain more insight on the problem under study, the *exact* expected total cost function (along with expected costs associated with repair and failure) is plotted in Figure 8 as a function of the time of inspection.
Based on the results presented in Figure 8, it can be noted that the optimal solution is a compromise between costs of repair and failure. That is, the optimal solution *does not* coincide with the maintenance schedule that minimizes the probability of failure, highlighting the importance of modeling repair activities explicitly, which correspond to partial damage states (Kupfer and Freudenthal, 1977). As it can be also noted from Figure 8, the time at which inspection is performed has a large impact in $E[C_T]$: in case inspections take place either too early or too late the effectiveness of maintenance activities is completely lost.

5. Conclusions

The purpose of this paper is to give an overview of the salient features of COSSAN-X in the context of the cycle management. COSSAN-X is a modular and flexible general purpose reliability software

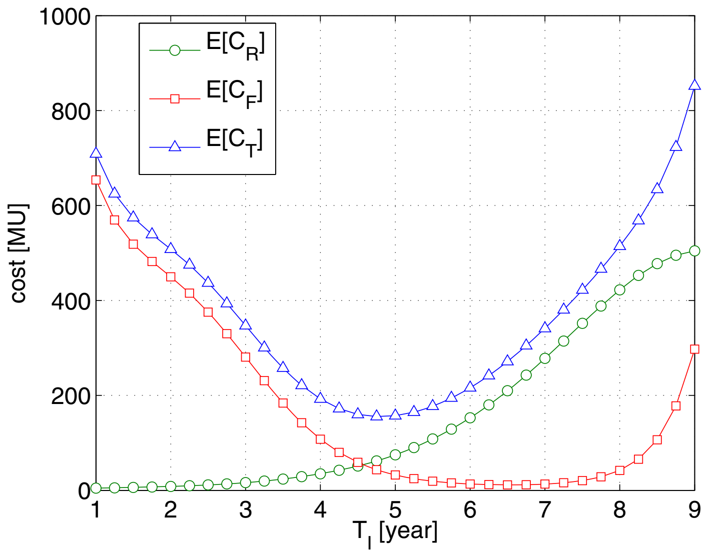

Figure 8. Expected total cost of operation ($E[C_T]$), expected costs associated with repair ($E[C_R]$) and failure ($E[C_F]$) events, respectively.

that allows the analyst to construct and define solution sequences specially tailored for the problem at hand.

COSSAN-X allows to perform life cycle management study coupling the 3rd party Finite Element analysis with the state of the art of the reliability and optimization algorithms. In this way the analyst can continue to use the deterministic FE-model that he is already familiar with. In fact, it appears difficult to otherwise adhere to the state-of-the-art, in terms of deterministic FE-modeling. It is recognized that one of the key difficulties associated with life cycle management and robust design are the extensive computational costs. COSSAN-X allows to reduce the computation costs adopting advanced and very efficient algorithms and parallelizing the analysis at different levels: relying on the parallelization within the deterministic FE code and the parallelization at the probabilistic analysis level.

The numerical example addressed in this contribution does not only demonstrate the capabilities of COSSAN-X but also highlights the importance of RBO in context with the design of an optimal maintenance schedule. The results indicate that the optimal maintenance strategy is not necessarily the one that minimizes the costs associated with failure; in fact, the optimal solution is a trade-off between different economical factors. Moreover, the time at which an inspection is performed plays a key role in the overall effectiveness of the maintenance strategy. Thus, inspection activities can be totally useless (although expensive) if scheduled at an inappropriate time.

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