Robustness Assessment for Progressive Collapse of Framed Structures using Pushdown Analysis Method

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Abstract: One primary goal of structural design is to avoid progressive collapse when encountered great earthquakes or other abnormal loads, so that it can leave enough time for escaping, protecting people’s lives and property. However, there is neither a uniform theory of structural robustness assessment nor a general methodology for quantification of the progressive collapse resistance of real complex structures.

In this paper, the residual reserve strength ratio is taken as a quantitative index to assess the robustness of frame structures. To obtain the progressive collapse resistance of damaged structures, a pushdown analysis method is developed herein, which considers the effects of the instant and duration of element removing as well as the locations of the removed elements. The proposed method is applied to a RC framed structure which is simulated by fiber section based beam elements with plastic hinges by OpenSees. It is demonstrated by this example that the approach is efficient and applicable for quantitative robustness assessment of complex structures in real world.

Keywords: Robustness, Progressive Collapse, Pushdown Analysis, Residual Reserve Strength Ratio

1. Introduction

Progressive collapse and robustness of structures under abnormal loads has been regarded as an important design consideration since the collapse of the Ronan Point Apartment building in 1968. Recently, interests in this topic have increased as a result of the terrorist bomb attack on the Murrah Federal Building in Oklahoma City in 1995 and the attack on the World Trade Center in New York in 2001. However, the considered accidental loads are mainly focused on explosions, fires, or terrorist attacks, while the problem of progressive collapse and robustness of generic buildings under earthquakes has yet been paid little attention to (Ellingwood, et al, 2005; Ellingwood and Dusenberry 2005; Kaewkulchai and Williamson 2006; Talaat 2007). The current seismic design codes (CEN 2002; China 2001) just specify some particular requirements and analysis approaches for preventing lateral incremental collapse mechanisms in which the building as a whole moves sideways, the vertical progressive collapse of damaged structures has been gained attentions only recently. Gurley (2008) confirmed that “records of earthquake damage show that earthquakes can also remove supports, often the corner columns causing two-way cantilever mechanism”, and emphasized that “earthquake engineering does need to include recognition of ‘lost column’ events and to incorporate design against progressive collapse”. The progressive collapse of vast buildings in the great Wenchuan earthquake in 2008 demonstrated once again that it is very important to take into account the ability of resisting progressive collapse and robustness of generic structures under rare earthquakes (Ye, et al, 2008).
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Although the robustness of structures in abnormal events has become a world wide research topic, there has been neither a uniform theory of structural robustness assessment nor a general methodology for quantification of the progressive collapse resistance of real complex structures. In this paper, a new pushdown analysis method is used to evaluate the residual capacity of damaged structures under rare earthquakes, which considers the effects of the duration and instant of element removing, the locations of the removed elements, the dynamic effects of undamaged bays, and the load steps in the analysis process. The residual reserve strength ratio (RRSR) is taken as a quantitative index to assess the robustness of damaged structures, and to quantify the progressive collapse resistance of real complex structures.

2. Pushdown Analysis Method

Vertical progressive collapse analysis of a structure is generally performed by the alternative load path (ALP) method, i.e., instantly removing one or several primary load-bearing elements, and then analyzing the structure’s remaining capability to absorb the damage (Japanese Society of Steel Construction 2007, Council on Tall Buildings and Urban Habitat 2007). There are four methods for progressive collapse analysis (PCA) (Marjanishvili and Agnew 2006): linear-elastic static, nonlinear static, linear-elastic dynamic and nonlinear dynamic methodologies. The linear static analysis procedure is performed using an amplified (usually by a factor of 2) combination of service loads, such as dead and live, applied statically, and response is evaluated by demand to capacity ratios (DCR). This analysis procedure is the simplest and easiest to perform. However, it is limited to relatively simple structures where both nonlinear effects and dynamic response effects can be easily and intuitively predicted. Dynamic analysis procedures (linear or nonlinear, especially nonlinear dynamic), although their accuracy is much higher, are usually avoided due to the complexity of the analysis. Additionally, evaluation and validation of the results can be very time-consuming. The nonlinear static analysis method implies a stepwise increase of amplified (by a factor of 2) vertical loads until maximum amplified loads are attained or until the structure collapses. This vertical pushover analysis procedure is often called “pushdown analysis method”. The advantage of this procedure is its ability to account for nonlinear effects, its usefulness in determining elastic and failure limits of the structure, and its ability of complementing the nonlinear dynamic analysis procedure. Therefore, the nonlinear static procedure is often recommended to be used in conjunction with nonlinear dynamic methodology as a supplemental analysis to determine the first yield and ultimate capacity limits, as well as to verify and validate dynamic analysis results. First yield and ultimate capacity of the structure can be used to determine and validate calculated ductility and rotations.

The pushdown analyses of a damaged structure can be accomplished in two different ways based on the loading way: uniform pushdown and bay pushdown. In the uniform pushdown analysis (Figure 1), gravity loads on the damaged structure are increased proportionally until the ultimate limit occurs. The failure may occur outside the damaged bays, and thus it might not be possible to estimate the residual capacity of the damaged bay. In the bay pushdown analysis (Figure 2), however, the gravity load is increased proportionally only in the bays that suffered damage until the ultimate limit is reached in the damaged bays (Dusenberry and Hamburger, 2006; Khandelwal and El-Tawil 2008; Kim and Park 2008; England, Agarwal and Blockley 2008; Kim and Kim 2009; among others). The residual capacity of the damaged bays can be measured in terms of the gravity overload factor calculated at instance of first failure in the damaged bays.
In this paper, bay pushdown analysis is used to analyze the effects of duration and instant of element removing as well as the locations of the removed elements, while uniform pushdown is used to investigate on the dynamic effects of the undamaged bays.

In pushdown analysis of a damaged structure, the vertical load is selected according to the provisions of General Services Administration (GSA 2003). The conventional pushdown analysis doesn’t consider the effects of the duration of element removing in the process of applying the vertical load, since the duration of element removing in static analysis will have no effects on the results of structural analysis. However, in practice the scheme of applying load does have effects on the analysis results in numerical simulation, i.e., if a very large load is applied instantaneously to a structure, there will be a serious convergence problem in the analysis program. In particular, it will be magnified when the initial error is accumulated to the stage in which the structure undergoes a strongly inelastic state. Therefore, the duration of element removing will have direct effects on the analyzed ultimate capacity of the structure in consideration.

In this paper, the conventional pushdown analysis procedure is revised to consider the effects of the duration of element removing by applying the unbalanced force to the remaining structure in sub-steps (Bao et al 2008; Kim and Kim 2008). As shown in Figure 3, the loading scheme of the pushdown analysis which considers the effects of the duration of element removing is as follows: firstly, apply the basic load \((D+0.25L)\) to the structure; secondly, remove the failed column, and apply the unbalanced load to the remaining structure in sub-steps; finally, apply the additional load \((D+0.25L)\) to the damaged bays until the completion of the loading or divergence of the program.

This procedure is equivalent to releasing the failed elements step by step. As shown in Figure 3c, the failed column is removed within the time interval \((t_0, t_1)\), if the duration of element removing is zero, then it means that the column fails instantaneously, which corresponds to a rectangular shock load; otherwise, it means that the failure of the element takes a short time, corresponding to a shock load that has an ascending segment.
Since it is not known that whether the load $2(D+0.25L)$ can be applied to the structure or not, in this paper, a load factor $\alpha$ is introduced to represent the load combination $\alpha(D+0.25L)$ that the structure can bear. The instability of the structure in the analysis procedure, i.e., the divergence of the program, is defined as the control criteria.

In static analysis approach specified by Unified Facilities Criteria (UFC 2005), it is allowed to remove the element firstly, and then to apply the basic load and the additional load. Therefore in this paper, the instant of element removing is also considered in the pushdown analysis. Considering the fact that there are two different loading ways in the static pushdown analyses, i.e., uniform pushdown and bay pushdown, the dynamic effects of the undamaged bay is also analyzed. In addition, the effects of the loading steps are considered in pushdown analysis.

In the pushdown analysis of this paper, the yielding of the rebar in tension means that the cross-section undergoes the initial plastic stage, corresponding to the yielding state of the structure; the cross section rotation is defined as peak (ultimate) rotation when the strain of core concrete at the edge reaches its ultimate strain, whose corresponding state is defined as the ultimate state; the state of instability of the structure or the divergence of the program is defined as collapse.

### 3. Quantitative Assessment of Structural Robustness

In fact, progressive collapse analysis is a part of robustness analysis. Progressive collapse of a structure refers to the condition when the failure of a local component (or localized region) leads to global system failure, and the final failure state of the structure is disproportionate to the triggering local failure. Structural robustness is defined as the ability of resisting progressive collapse, and it indicates the overall performance of the damaged structure assuming a load-bearing element removed (Faber et al 2006; Baker, Schubert and Faber 2008). The progressive collapse of the structure is more likely to happen for lack of robustness.

The current progressive collapse analysis procedures can only give a qualitative assessment of the robustness of the overall structural system. However, to conduct a progressive collapse control design, we need to know the quantitative results of structural robustness and the reserve load-bearing capacity of the damaged structure. In this paper, the quantitative value of the robustness of a structure is obtained by a redundancy index:

$$ R = \frac{P_{\text{damaged}}}{P_{\text{design}}} $$ (1)
where $R$ is the residual reserve strength ratio (RRSR); $P_{\text{damaged}}$ is the ultimate load of the damaged structure; $P_{\text{design}}$ is design load of the intact structure.

This redundancy index, which is proposed by Lalani and Shuttlewoeth (1990), can tell one whether there are enough alternate paths to safely transfer the loads originally resisted by the failed components. The residual reserve strength ratio can be calculated in the following steps: firstly, $P_{\text{design}}$ is calculated, because the results of dynamic analysis are more realistic, it is regarded as the reference, and the maximum vertical displacement is obtained by dynamic analysis, in which the vertical load is selected according to the provisions of GSA, as shown in Figure 4; secondly, pushdown analysis approach is applied to the structure, and a pushdown curve is obtained, $P_{\text{design}}$ is the load factor corresponding to the maximum vertical displacement which is obtained by dynamic analysis on the pushdown curve; finally, determine $P_{\text{damaged}}$, which is the load factor corresponding to the ultimate load-bearing capacity of the structure on the pushdown curve.

![Figure 4. Schematic of load applying in vertical dynamic analysis](a)

4. Case Study: Robustness Assessment for Progressive Collapse of RC Framed Structures

To investigate on the robustness for progressive collapse of the codified designed structures, a typical 3-bay and 5-storey reinforced concrete moment frame structure is designed according to the current seismic design code of buildings (China, 2001). The plan and elevation of the model structure and the details of structural members are shown in Figure 5. The vertical loads are listed in Table I. The mechanical properties of the materials used in the structure are: compressive strength of concrete $f_{c}^{' } = 29.76\text{MPa}$, yield strength of reinforcement $f_{y} = 388.0\text{MPa}$, the Young’s modulus of concrete and steel are $E_{c} = 3.356 \times 10^{4}\text{MPa}, E_{s} = 200\text{GPa}$, respectively. The first three modal periods of the structure are shown in Table II.

The OpenSees is chosen as the simulation platform. Concrete is modeled by Concrete01, confinement is specified implicitly by using the confined stress-strain relationships proposed by Mander, Priestley and Park (1988); reinforcing steel is modeled by Steel02 with 1% strain hardening, the material properties used in the simulations are listed in Table III. Beams and columns are modeled by BeamWithHinges elements with co-rotational geometric transformation, two fibers in the cover region and ten fibers in the confined core are adequate to achieve a stable, converged subassembly response, and the length of plastic hinge length is defined as the height of the section of the beam.
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![Diagram](https://via.placeholder.com/150)

*Figure 5. Plan and Elevation of the RC frame and details of structural members (mm)*

Note: C1 represents a column in the first story in column line C; AB2 represents a beam on the second floor in bay AB.

<table>
<thead>
<tr>
<th>Locations</th>
<th>Uniformly distributed loads (kN/m)</th>
<th>Concentrated loads (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dead loads</td>
<td>Live loads</td>
</tr>
<tr>
<td>Roof floor</td>
<td>12.09</td>
<td>1.10</td>
</tr>
<tr>
<td>Middle floor</td>
<td>9.35</td>
<td>4.40</td>
</tr>
<tr>
<td>Ground floor</td>
<td>9.35</td>
<td>4.40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>directions</th>
<th>Before the gravity load is applied (s)</th>
<th>After the gravity load is applied (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The first mode</td>
<td>The second mode</td>
</tr>
<tr>
<td>X</td>
<td>0.737</td>
<td>0.219</td>
</tr>
<tr>
<td>Y</td>
<td>0.059</td>
<td>0.053</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Types</th>
<th>Parameters of concrete material properties (C35)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconfined</td>
<td>Peak stress</td>
</tr>
<tr>
<td>Confined column</td>
<td>-29.76</td>
</tr>
<tr>
<td>Confined beam</td>
<td>-37.65</td>
</tr>
<tr>
<td></td>
<td>-36.20</td>
</tr>
</tbody>
</table>

Note: The unit of stress is MPa.

4.1. THE EFFECTS OF DURATION OF ELEMENT REMOVING

It is specified that the duration of element removing is at least less than 1/10 of the vibration period of the structure in the progressive analysis by GSA (2003). Since the vibration period of the structure herein is 0.945s, so 0.10s is selected as the duration of element removing.

The pushdown analysis considering the effects of the duration of element removing is applied to the model frame. The analysis results are shown in Table IV. The failure of the structure is represented by the
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occurrence of the plastic hinges at the ends of the beams, and the collapse of the structure is a consequence of the instability of the system.

Table IV. The results of pushdown analysis considering the effects of the duration of element removing

<table>
<thead>
<tr>
<th>Locations of cross sections</th>
<th>$\alpha_y$</th>
<th>$\theta_y$ (rad)</th>
<th>$\alpha_u$</th>
<th>$\theta_u$ (rad)</th>
<th>$\theta_{\text{max}}$ (rad)</th>
<th>$\theta_u / \theta_y$</th>
<th>$\theta_{\text{max}} - \theta_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right side of beam BC1</td>
<td>1.01</td>
<td>0.00317</td>
<td>1.49</td>
<td>0.04171</td>
<td>0.17817</td>
<td>13.16</td>
<td>0.03854</td>
</tr>
<tr>
<td>Left side of beam BC2</td>
<td>1.04</td>
<td>0.00272</td>
<td>1.47</td>
<td>0.03515</td>
<td>0.17363</td>
<td>12.94</td>
<td>0.03243</td>
</tr>
<tr>
<td>Right side of beam BC2</td>
<td>1.04</td>
<td>0.00279</td>
<td>1.48</td>
<td>0.03798</td>
<td>0.17603</td>
<td>13.61</td>
<td>0.03519</td>
</tr>
<tr>
<td>Left side of beam BC1</td>
<td>1.06</td>
<td>0.00276</td>
<td>1.47</td>
<td>0.03779</td>
<td>0.17243</td>
<td>13.67</td>
<td>0.03503</td>
</tr>
<tr>
<td>Left side of beam BC3</td>
<td>1.10</td>
<td>0.00275</td>
<td>1.47</td>
<td>0.03528</td>
<td>0.17508</td>
<td>12.83</td>
<td>0.03253</td>
</tr>
<tr>
<td>Right side of beam BC3</td>
<td>1.10</td>
<td>0.00283</td>
<td>1.48</td>
<td>0.03778</td>
<td>0.17529</td>
<td>13.33</td>
<td>0.03495</td>
</tr>
<tr>
<td>Left side of beam BC4</td>
<td>1.12</td>
<td>0.00273</td>
<td>1.47</td>
<td>0.03524</td>
<td>0.17561</td>
<td>12.89</td>
<td>0.03251</td>
</tr>
<tr>
<td>Right side of beam BC4</td>
<td>1.12</td>
<td>0.00281</td>
<td>1.48</td>
<td>0.03773</td>
<td>0.17527</td>
<td>13.43</td>
<td>0.03492</td>
</tr>
<tr>
<td>Left side of beam BC5</td>
<td>1.17</td>
<td>0.00283</td>
<td>1.48</td>
<td>0.03800</td>
<td>0.17536</td>
<td>13.44</td>
<td>0.03517</td>
</tr>
<tr>
<td>Right side of beam BC5</td>
<td>1.18</td>
<td>0.00292</td>
<td>1.48</td>
<td>0.03685</td>
<td>0.17436</td>
<td>12.64</td>
<td>0.03393</td>
</tr>
<tr>
<td>Left side of beam AB1</td>
<td>1.26</td>
<td>0.00346</td>
<td>1.65</td>
<td>0.04167</td>
<td>0.08142</td>
<td>12.06</td>
<td>0.03822</td>
</tr>
<tr>
<td>Left side of beam AB2</td>
<td>1.26</td>
<td>0.00326</td>
<td>1.64</td>
<td>0.03674</td>
<td>0.07578</td>
<td>11.29</td>
<td>0.03349</td>
</tr>
<tr>
<td>Left side of beam AB3</td>
<td>1.26</td>
<td>0.00326</td>
<td>1.64</td>
<td>0.03576</td>
<td>0.07379</td>
<td>10.95</td>
<td>0.03249</td>
</tr>
<tr>
<td>Left side of beam AB4</td>
<td>1.27</td>
<td>0.00329</td>
<td>1.65</td>
<td>0.03644</td>
<td>0.07273</td>
<td>11.09</td>
<td>0.03316</td>
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<tr>
<td>Right side of beam AB1</td>
<td>1.33</td>
<td>0.00495</td>
<td>1.68</td>
<td>0.04324</td>
<td>0.07803</td>
<td>8.74</td>
<td>0.03830</td>
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<tr>
<td>Right side of beam AB2</td>
<td>1.33</td>
<td>0.00475</td>
<td>1.65</td>
<td>0.03852</td>
<td>0.07642</td>
<td>8.10</td>
<td>0.03376</td>
</tr>
<tr>
<td>Right side of beam AB3</td>
<td>1.33</td>
<td>0.00474</td>
<td>1.65</td>
<td>0.03750</td>
<td>0.07458</td>
<td>7.91</td>
<td>0.03276</td>
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<tr>
<td>Right side of beam AB4</td>
<td>1.33</td>
<td>0.00471</td>
<td>1.66</td>
<td>0.03840</td>
<td>0.07394</td>
<td>8.16</td>
<td>0.03369</td>
</tr>
<tr>
<td>Left side of beam AB5</td>
<td>1.34</td>
<td>0.00299</td>
<td>1.67</td>
<td>0.03682</td>
<td>0.07033</td>
<td>12.32</td>
<td>0.03384</td>
</tr>
<tr>
<td>Right side of beam AB5</td>
<td>1.35</td>
<td>0.00534</td>
<td>1.67</td>
<td>0.03960</td>
<td>0.07401</td>
<td>7.42</td>
<td>0.03427</td>
</tr>
<tr>
<td>Bottom of column A1</td>
<td>1.48</td>
<td>0.00371</td>
<td>-</td>
<td>-</td>
<td>0.01589</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Top of column A1</td>
<td>1.67</td>
<td>0.00335</td>
<td>-</td>
<td>-</td>
<td>0.00648</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The mean value of $\theta_u - \theta_y$ is 0.0345, the standard deviation is 0.0019, and the coefficient of variation is 0.055.

Note: $\alpha_y$, $\alpha_u$ are the load factors corresponding to the first yielding point and the ultimate point respectively; $\theta_y$, $\theta_u$ are the corresponding rotations; $\theta_{\text{max}}$ is the maximum rotation in the analysis.

The results presented in Table IV indicate that, the initial yield rotations of different elements are not the same. It is about $0.0027 - 0.0029$ rad at the left end of beam BC, $0.0027 - 0.0032$ rad at the right end of beam BC, $0.032 - 0.034$ rad at the left end of beam AB, and $0.047 - 0.049$ rad at the right end of beam AB. However, there is no obvious difference in the ultimate rotations at the ends of different elements, although the ultimate rotations at both ends of beam AB1 are larger than others. The maximum rotation in the analysis has relations with the span of the beam, and it has nothing to do with the locations of the cross-sections, the section-rotation of beam BC is the largest in the process of analysis, it is greater than 0.17 rad, although the maximum section-rotation of beam AB maintains at 0.07 rad. Plastic hinge occurs only in the column A1 in the whole process of the analysis, and the maximum rotation of the column maintains at a low level, so the loss of a single column will not lead to the complete collapse of the structure.
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Although $\theta_i/\theta_j$ at the right end of beam AB is different from the others, the initial yield rotation of each element is small, the ultimate plastic rotation $\theta_u - \theta_i$ of each element is close to each other, and the standard deviation and coefficient of variation of them are also small. So, although the initial yield rotation of each element is related with the location of the cross-section, the ultimate plastic rotation of each element (which is corresponding to the “a” in Federal Emergency Management Agency) (FEMA, 2000) is close to each other, and it is larger than that is recommended in FEMA (0.0345 rad > 0.025 rad). The deformation limits of the rotation degrees of components obtained in this paper is close to the limit value recommended in the UFC (0.105 rad), but it is larger than that offered by FEMA (0.05 rad).

- The point beyond the yielding state
- The point beyond the ultimate state
- The point beyond the collapse state

(a) Distribution of plastic hinges of the damaged structure when there is a rotation beyond the yielding state
(b) Distribution of plastic hinges of the damaged structure when there is a rotation beyond the ultimate state
(c) Distribution of plastic hinges of the damaged structure in instability state considering the effects of the duration of element removing in analysis
(d) Distribution of plastic hinges of the damaged structure in instability state without considering the effects of the duration of element removing in analysis

Figure 6. The effects of the duration of element removing on the failure modes of the damaged structures
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As shown in Figure 6 and Figure 7, the duration of element removing has no effects on the instant of the occurrence of structural yielding rotation angle, ultimate rotation angle and the rotation angle against collapse for the first time (it is corresponding to the load factor in pushdown analysis), the two pushdown curves almost coincides when the load factor is less than 1.69 in Figure 7. If we don’t consider the effects of duration of element removing, the structure will collapse instantly (it is corresponding to the load factor of 1.72) once there is a rotation angle of members which exceeds the limit value against collapse (it is corresponding to the load factor of 1.69). Otherwise, a larger ultimate load-bearing ability of the structure can be obtained, the structure collapse when the load factor is 1.92, although the rotation of beam BC is far more than the limit value against collapse at the moment. Therefore, a greater structural robustness will be gained when we consider the effects of duration of element removing, although it has no effects on the failure modes.

4.2. THE EFFECTS OF THE INSTANT OF ELEMENT REMOVING

In the static analysis approach specified by Unified Facilities Criteria (UFC 2005), it is allowed to remove the element firstly, and then to apply the basic load and the additional load. Therefore in this paper, two schemes of the pushdown analysis are considered. In the first case, the element is removed after the gravity load is applied to the structure in pushdown analysis; while in the second case, the element is removed before the gravity load is applied to the structure, which is equivalent to the consideration of the initial deformation in the analysis. The pushdown curves are given in Figure 8, and the distributions of plastic hinges of the damaged structure are shown in Figure 9.

Figure 7. Pushdown curve of the model structures considering the effects of the duration of element removing

Figure 8. Pushdown curves of the damaged structures considering the effects of the instant of element removing
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As shown in Figures 8 & 9, a greater ultimate ability of resisting progressive collapse of the structure is obtained if an element is removed after the gravity load is applied to the structure in pushdown analysis, and that it has no effects on the failure modes. The structure will collapse due to the large plastic rotation angles at the ends of beams in both schemes of pushdown analysis.

4.3. THE DYNAMIC EFFECTS OF UNDAMAGED BAYS

The uniform pushdown is used to investigate on the dynamic effects of the undamaged bays, the distributions of plastic hinges of the damaged structure are shown in Figure 10.
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(a) Distribution of plastic hinges of the damaged structure when there is a cross-section rotation beyond the ultimate limit firstly in bay pushdown analysis

(b) Distribution of plastic hinges of the damaged structure when there is a cross-section rotation beyond the limit against collapse firstly in bay pushdown analysis

(c) Distribution of plastic hinges of the damaged structure in instability state in bay pushdown analysis

(d) Distribution of plastic hinges of the damaged structure when there is a rotation beyond the ultimate limit firstly in uniform pushdown analysis

(e) Distribution of plastic hinges of the damaged structure when there is a element rotation beyond the limit against collapse firstly in uniform pushdown analysis

(f) Distribution of plastic hinges of the damaged structure in instability state in uniform pushdown analysis

Figure 10. The dynamic effects of undamaged bays on the ability of resisting progressive collapse of the damaged structures

As shown in Figure 10, it has no effects on the failure modes of the structure by considering the dynamic effects of the undamaged bays, but a larger ultimate load factor is obtained in the analysis. This can be explained as follows: the additional load applied on the undamaged bays provides a beneficial effect on the capacity of beam CD, while the bearing moment caused by the additional load is not so much.

4.4. THE EFFECTS OF LOAD STEPS
In pushdown analysis of a damaged structure, the vertical loads can be applied to the structure in different load steps. The effects of the load steps are also considered here, the results are shown in Table V.

<table>
<thead>
<tr>
<th>Load step</th>
<th>Considering the effects of the duration of element removing</th>
<th>Not considering the effects of the duration of element removing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>1.84</td>
<td>1.78</td>
</tr>
<tr>
<td>0.005</td>
<td>1.98</td>
<td>1.85</td>
</tr>
<tr>
<td>0.01</td>
<td>1.92</td>
<td>1.72</td>
</tr>
<tr>
<td>0.02</td>
<td>0.84</td>
<td>2.00</td>
</tr>
<tr>
<td>0.05</td>
<td>0.80</td>
<td>0.80</td>
</tr>
</tbody>
</table>

It is observed from Table V that, the failure modes of different analysis steps are the same, but the ultimate load factors are greatly different from each other, a smaller ultimate load factor is gained when the load step is too large or too small, the differences may come from some singular points whose occurrence are random in the analysis. However, different cases demonstrate complete consistency until there is a cross-section rotation which is beyond the limit against collapse occurred firstly, so the deformation limits of the rotation degrees of components for reinforced concrete obtained in this analysis is consistent with that given in UFC. The limit of rotation degree of components is suggested as the major control criteria for the failure of components or for preventing the structure from collapse.

### 4.5. Quantitative Analysis of Structural Robustness

The pushdown analysis is performed on the damaged structure which assumes that the column in column line B on each floor is removed respectively. The robustness of the damaged structure is obtained using the methods described above. For example, the response of the frame with column B1 removed is analyzed. A load factor of 1.305 corresponding to the peak vertical displacement which is obtained by nonlinear dynamic analysis is obtained in the pushdown curve, and 1.305 is taken as the nominal load, the load factor of 1.72 is obtained when structural instability occurred. So, the robustness of the damaged structure is calculated as:

\[
R = \frac{P_{\text{damaged}}}{P_{\text{design}}} = \frac{1.72}{1.305} = 1.32
\]

![Figure 11](image_url) The robustness indices of the damaged structures assuming the columns in column line B removed for each floor respectively
The robustness indices of the damaged structure for column B1, B2, B3, B4, B5 removal cases are shown in Figure 11. For column B3 removal case, the impact on the overall performance of the structure is the smallest; while for column B2 removal case, it is the largest. If the smallest value is regarded as the robustness of the whole building, then it is 1.19. However, the final robustness of the structure needs to be further studied due to the different probability of occurrence of different cases.

5. Conclusions

The quantitative assessment of the robustness for vertical progressive collapse of framed structures are made thorough investigations on using a new pushdown analysis method in this paper, the following conclusions are drawn through this research:

1. Pushdown analysis considering the effects of the instant and duration of element removing as well as dynamic effects of undamaged bays is performed, it is founded that they all have no effects on the failure modes of the structure, but they do have influences on the ultimate ability of resisting progressive collapse of the structure. A greater ultimate ability of the structure will be obtained while considering the effects of the duration of element removing and dynamic effects of undamaged bays; and a greater ultimate ability of the structure will also be obtained if the component is removed before the gravity load is applied to the structure.

2. A smaller ultimate load factor is obtained if the load step is too large or too small. However, different load steps have no effects on the failure modes of the structure, the pushdown curves demonstrate complete consistency until there is a cross-section rotation which is beyond the limit against collapse occurs firstly. Accordingly, the deformation limits of the rotation degrees of component are suggested as the major criteria for the failure of the components or preventing the structure from collapse.

3. The failure modes can be efficiently determined by the pushdown analysis of a structure, and the robustness of the structure can be quantitatively assessed by the residual reserve strength ratio. It is hoped that the method proposed by this paper will provide a practical method that the engineers could use to design more robust buildings.

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