

Damage Factor Estimation of Crane-Hook (A Database Approach with Image, Knowledge and Simulation)

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Abstract: The main topic of this study is a damage factor estimation of crane-hooks. Our estimation is to recognize the tendency of the load condition. We examine the relation between the load condition and its deformation by numerical analysis based on FEM. The relation is recorded into the database. The feature points are detected from the crane-hook images where these points are defined in order to correspond to the numerical analysis results in the database. These points are compared to the relation that the database has. A data is defined as the force that minimizes the difference between the feature points of the image and the deformed nodal position of the database. Using Bayesian theory, we estimate the distribution of the load condition in case of that the crane-hooks are damaged. We find out some knowledge about the crane-hooks from the estimation result. These are as follows: the load condition lies between the most downward point and the tip-end point, the direction is toward the gravity direction.

Keywords: Damage factor, Database approach, Bayesian theory, Crane-hook

1. Introduction

Recently, excavators having a crane-hook are widely used in construction works site. One reason is that there are work sites where the crane trucks for suspension work are not available because of the narrowness of the site; an excavator has superior maneuverability than a crane truck in general. Another reason is that such an excavator is convenient since they can perform the conventional digging tasks as well as the suspension works mentioned above. However, there are cases that the crane-hooks are damaged during some kind of suspension works. From the view point of safety, such damage must be avoided. Identification of the cause of the damage is one of the key points toward the safety improvement. In our study, an estimation approach of mechanical damage factor of the crane-hooks is conducted based on obtained examples of the damaged crane-hook.

The crane-hook attached to the excavators experiences various forces. We estimate the load conditions that are assumed to be crucial to the crane-hook damages. We construct an FEM model of the crane-hook referring to one of its actual designs. A database is prepared based on the FEM model; it is constructed as a collection of a number of various possible load conditions and the corresponding deformation values

obtained as the results of the FEM analysis. The database is used to identify the load conditions that were fatal to those damaged crane-hooks. Some of the feature points are selected on the crane-hook design; the deformation of a damaged crane-hook can be then obtained based on the feature points detected by means of the image processing. The critical load condition of the damaged crane-hook is calculated by comparing the obtained actual deformation and the simulated deformation values in the database. On the basis of these calculated load conditions, the critical load condition for the crane-hook is estimated as a statistical distribution based on the Bayesian approach.

The outline of this paper is as follows. In chapter 2, we construct the model of crane-hooks using FEM analysis and introduce the Load-Deformation (L-D) database. In chapter 3, we explain the image processing of the damaged crane-hooks in order to obtain the feature points. In chapter 4, we apply the Bayesian approach to the critical load condition estimation. Example damage factor estimation is conducted based on the actual damaged crane-hooks in chapter 5. Some concluding remarks and future works are expressed in chapter 6.

2. Crane-hook Model and Load-Deformation Database

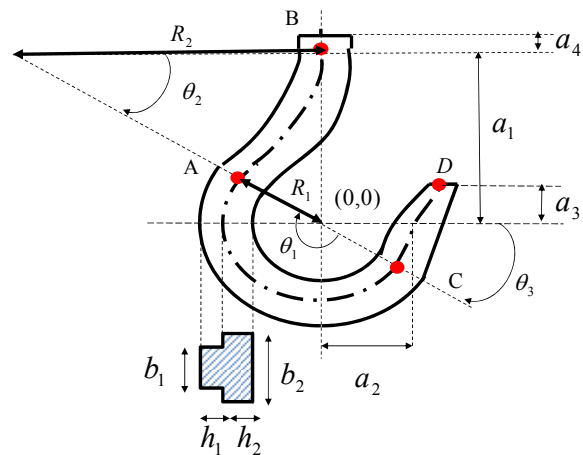
2.1. CRANE-HOOK MODEL

We construct an FEM model of the crane-hooks based on one of its actual design. Figure 2.1(a) shows the crane-hook adopted as the reference. The shape of the crane-hook model is shown in figure 2.1(b). As indicated in the figure 2.1(b), the latch part is omitted in the adopted model because it does not contribute to supporting an applied load. The dimensions of the crane-hook dealt with in this study are indicated in the figure 2.1(b). The contour shape of the crane-hook is represented as follows:

- Straight line from the tip-end point D to point C
- Circle of radius R_1 from point C to point A
- Circle of radius R_2 from point A to point B



(a)



(b)

Figure 2.1 Crane-Hook (a) and its Model (b)

Table 2.2. The values of dimension parameters					
Parameter	Value	Parameter	Value	Coordinate	Value
a_1	90[mm]	h_1	16[mm]	θ_3	30[degree]
a_2	44[mm]	h_2	16[mm]	A	(-38.1,22)[mm]
a_3	10[mm]	R_1	44[mm]	B	(0,90)[mm]
a_4	10[mm]	R_2	99[mm]	C	(38.1,-22)[mm]
b_1	16[mm]	θ_1	180[degree]	D	(54,10)[mm]
b_2	32[mm]	θ_2	30[degree]		

As shown in the lower left in figure 2.1 (b), the T-shape cross-section is adopted. Table 2.2 shows the values of the dimension parameters adopted in this study.

On the basis of the adopted crane-hook shape, an FEM model is developed based on the 2D beam finite element. The crane-hook is divided into N_{node} elements; the stiffness equation for i th element is expressed as

$$\mathbf{F}_i = \mathbf{K}_i \mathbf{U}_i \tag{2.1}$$

where \mathbf{F}_i , \mathbf{K}_i and \mathbf{U}_i are the force vector, stiffness matrix and displacement vector referring the element coordinate. The adopted element stiffness matrix[2] is

$$\mathbf{K}_i = \frac{E}{l_i^3} \begin{bmatrix} A_i l_i^2 & 0 & 0 & -A_i l_i^2 & 0 & 0 \\ & 12 I_i & 6 l_i I_i & 0 & -12 I_i & 6 l_i I_i \\ & & 4 l_i^2 I_i & 0 & -6 l_i I_i & 2 l_i^2 I_i \\ & & & A_i l_i^2 & 0 & 0 \\ Sym . & & & & 12 I_i & -6 l_i I_i \\ & & & & & 4 l_i^2 I_i \end{bmatrix}, \tag{2.2}$$

where l_i is i th element length, A_i is i th element sectional area, I_i is the geometrical moment of inertia of i th element cross-section and E is Young's modulus of the material. The corresponding displacement and force vectors are respectively,

$$\mathbf{U}_i = [x_i, z_i, \theta_i, x_{(i+1)}, z_{(i+1)}, \theta_{(i+1)}]^T, \tag{2.3}$$

$$\mathbf{F}_i = [x f_i, y f_i, \theta f_i, x f_{(i+1)}, y f_{(i+1)}, \theta f_{(i+1)}]^T, \tag{2.4}$$

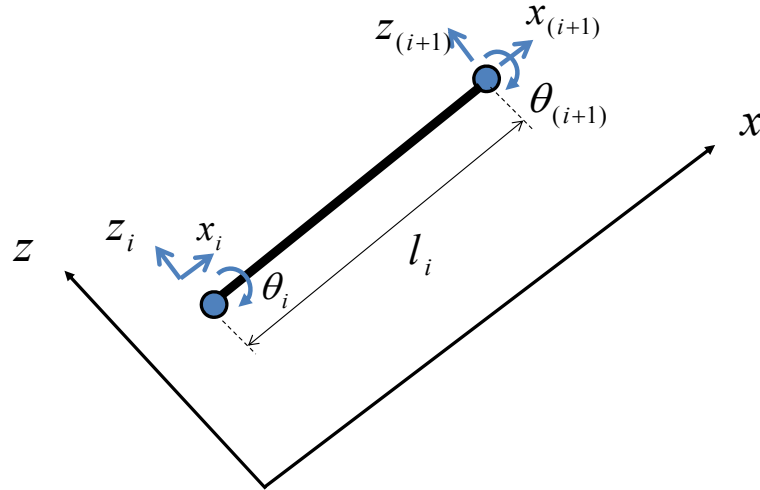


Figure 2.3 i th 2D Beam Finite Element's Coordinate

where x_i and $x_{(i+1)}$ are the displacement of x direction of both edges of i th element, z_i and $z_{(i+1)}$ are the displacement of z direction of both edges of i th element, θ_i and $\theta_{(i+1)}$ are the deflection angle of both edges of the i th element. The subscript of the force elements correspond to the direction of the displacement respectively. Figure 2.3 shows i th element coordinate.

The following stiffness equation of FEM model of the crane-hook is obtained based on the coordinate transformation and superposition of element stiffness equation (2.2)

$$\mathbf{F} = \mathbf{K}\mathbf{U} \quad (2.5)$$

where $\mathbf{F} = [x f_1, z f_1, \theta f_1, x f_2, z f_2, \theta f_2, \dots, x f_N, z f_N, \theta f_N]^T$ and $\mathbf{U} = [x_1, z_1, \theta_1, x_2, z_2, \theta_2, \dots, x_N, z_N, \theta_N]^T$. On the basis of the equation, the deformation of the crane-hook corresponding to an assumed load is calculated. In this study, the load and obtained corresponding deformation values are to be recorded in a database explained in the following section.

2.2 L-D DATABASE

In order to estimate the load condition that caused the damage on a crane-hook, we use the digital image of the damaged crane-hook and L-D database. The L-D database is designed to have the following information obtained based on the stiffness equation (2.5):

- Assumed force on the crane-hook
- Corresponding deformation
- Position that is obtained by dividing the sum of all deformation by the element number (Averaged position)

Force pattern adopted for the preparation of the L-D database is indicated in table 2.4. The indicated L-D database seems to have excessive information since the adopted FEM analysis is only linear in the current

study we are considering FEM analysis taking account of non linearity of the material property such as plasticity in the future work; the structure of the database is designed from the generality point of view.

The LD database consists of two tables; one is for the analysis conditions and the other is for the analysis results. Two tables are related with the analysis number that is unique to each of the analysis. Figure 2.5 shows the entities and its attributes of the LD database.

The analysis is performed based on the “analysis condition” given in the condition table and generates the corresponding result to be recorded in the resultant table. On the contrary, the estimation of the fatal load condition of the damaged crane-hook is performed based on the deformation information in the resultant table and the corresponding force is identified by means of error minimization approach. The detail is discussed in the next section.

Number of applied positions	30	(equally-spaced interval)
Magnitude(kN)	10~200	(10kN interval)
Direction(degree)	-90~90	(interval 10degree, 0 for downward direction)

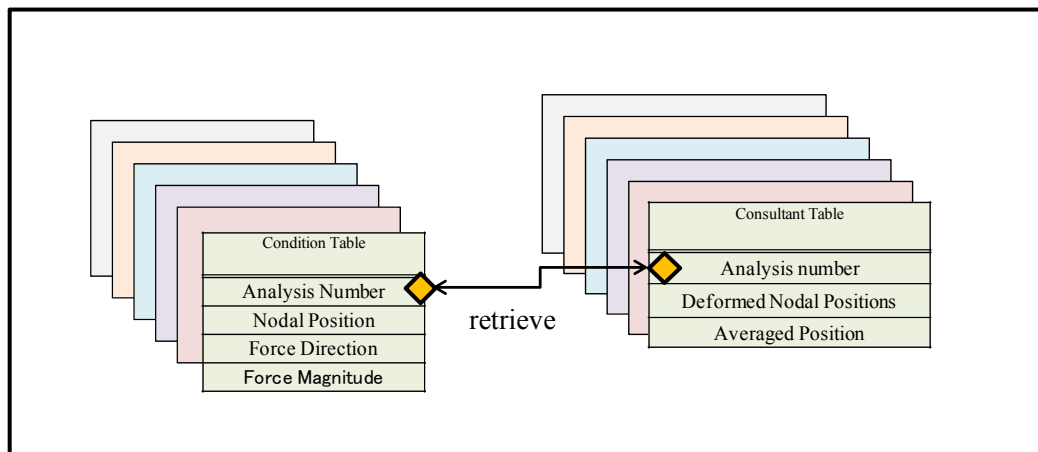


Figure 2.5 Database Entities and Attributes

3. Feature Points Detection from Crane-Hook Images

In order to estimate the damage factor of crane-hooks, we use their digital images. We define a number of feature points on crane-hook. The crane-hook images are preprocessed by means of such as filtering and edge detection; the feature points are identified based on the obtained crane-hook outline image. The overview of the flowchart of the feature points detection process is shown in figure 3.1.

An obtained crane-hook image is evaluated based on the deformation data in the L-D database. The evaluation is performed through the feature points that are set on the crane-hook image, where the feature points are the points that divide the crane-hook image into N parts. The image processing and feature points detection are described in the following sections.

3.1. PREPROCESSING OF CRANE-HOOK IMAGE

In order to detect the feature points mentioned above, the digital image of a damaged crane-hook is processed to obtain its shape outline. First, the edge preserving smoothing[3] is applied to the image. This image filtering process can eliminate noise without shading off the edges. This filtering has the property that the edges are sharpened. It is efficient for edge detection to process this filtering. On the basis of the filtering process, the image shown in figure 3.2(b) is obtained from the original image in figure 3.2(a). Second, the edge detection process by means of the sobel operator[4] is performed on the image. The process is strong against the noise and this is the most generally used. On the bases of the process, figure 3.2(c) is obtained from the filtered image (figure 3.2(b)). Finally, the outline tracing is conducted on the image to obtain the shape of the damaged crane-hook. Figure 3.2(d) shows the obtained outline of the crane-hook shape based on the original image shown in figure 3.2(a). The outlines coming from the residual noise marks are ignored since only largest outline is selected as the significant part at this stage.

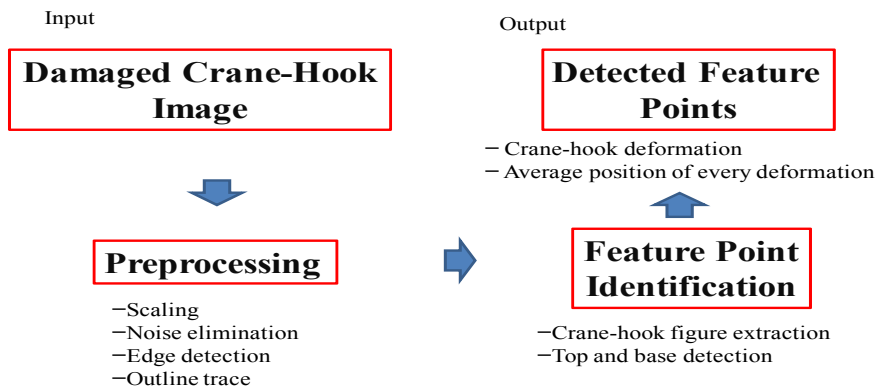
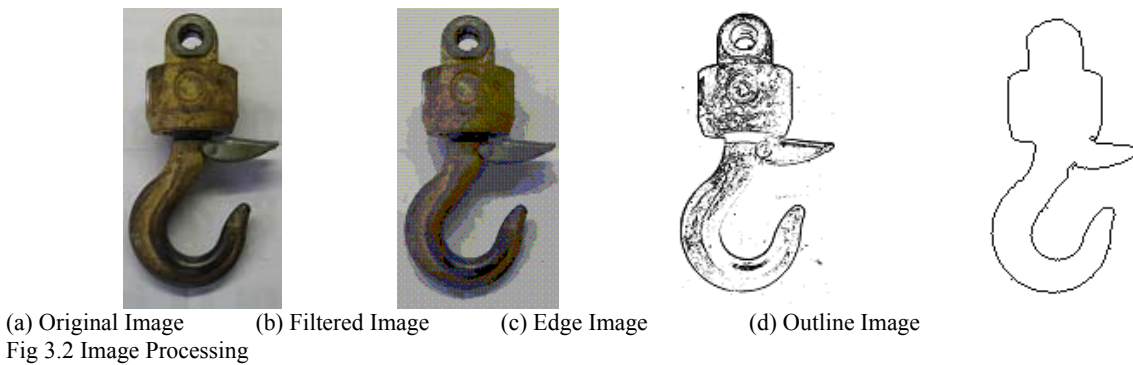


Figure 3.1 Overview of Feature Points Detection Process



3.2. DETECTING FEATURE POINTS

3.2.1 Tip-end and base locations

In this section, we express how to detect the tip-end and base locations on the crane-hook images. Figure 3.3 and 3.4 show the magnified image of the tip-end base parts, respectively.

- The tip-end location (Figure 3.3)

The crane-hook image that is obtained by above processing is divided at a ratio of 3:2 by the horizontal line. The most right side pixel of the lower divided image is regarded as the tip-end location of the crane-hook. More under pixel is the tip-end location of the crane-hook in case of more than one existence of the most right side pixel.

- The two base locations(Figure 3.4)

Trace the both sides of the obtained outline from the tip-end point with a constant interval d . Detect acute positions of the both sides of tracing that the angle θ (figure3.5) between the adjacent tracing line segments is such that $\theta > \underline{\theta}$; such positions are determined as the base points. The threshold angle $\underline{\theta}$ is 60 degree in this study.

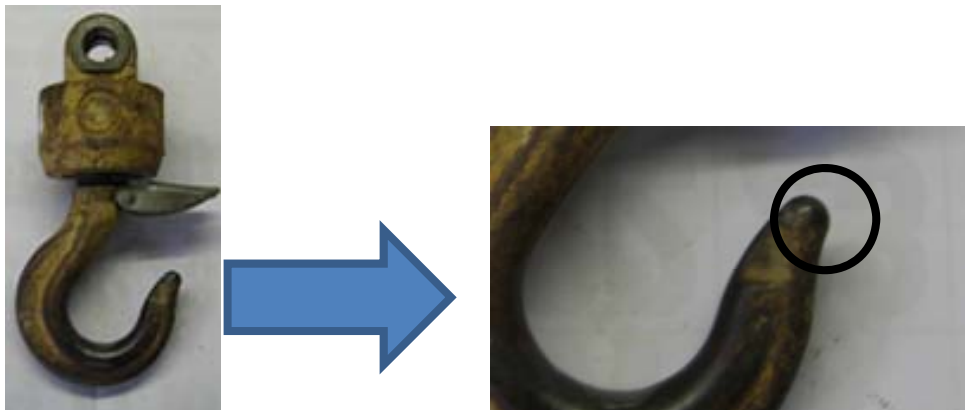


Figure 3.3 Crane-hook Tip-end

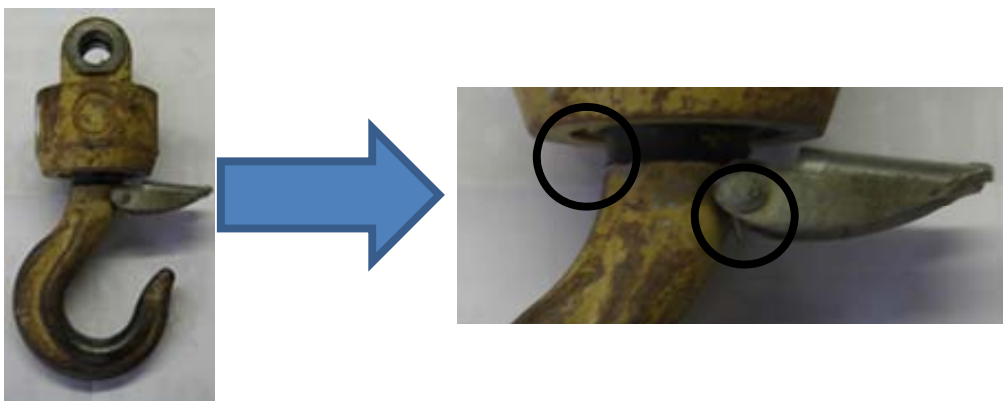


Figure 3.4 Crane-hook Bases

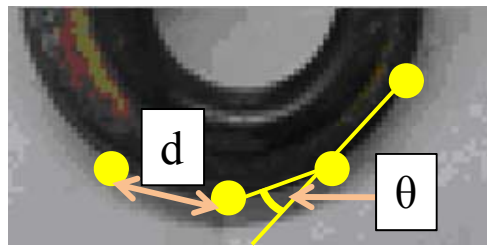


Figure 3.5 Searching Acute Position

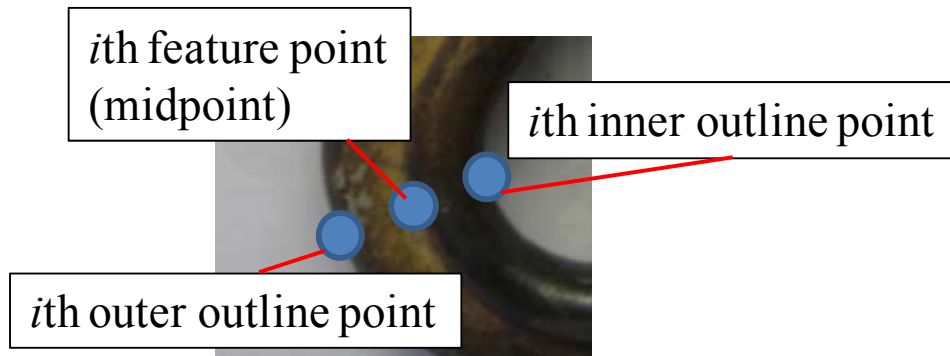


Figure 3.6 Feature Point

3.2.2. Feature Points

Take N_{feat} points respectively on the both of the inner and outer outlines of the hook part from the tip-end point to the base points at a constant interval; in other words, the inner and outer outlines are divided into $N_{feat} - 1$ parts, respectively. The midpoints of the corresponding points on the inner and outer outlines obtained like this are selected as the feature points (figure3.6). These feature points are used for comparison with the L-D database as discussed in the following.

4. Damage Estimation Using Bayesian Theory

4.1. PROBABILISTIC MODEL

We estimate the position and direction of the critical load condition of the damaged crane-hook as the damage factor in this study. The magnitude of the load condition is currently not taken into consideration because the current linear FEM analysis can not determine the accurate magnitude of the applied loading condition in the case of nonlinear or plastic deformation that causes structural damage in practical situation. The load position α_p and the direction α_d are dealt with as random variables, where α_p donates the coordinate along with the mid-curve of the inner and outer outlines. The probability density functions of

these random variables are assumed to be normal distribution in such a case because of the central limit theorem[5]. They are expressed as

$$\begin{cases} g_p(\alpha_p) = \frac{1}{\sqrt{2\pi}\sigma_p} \exp\left(-\frac{(\alpha_p - \mu_p)^2}{2\sigma_p^2}\right) \\ g_d(\alpha_d) = \frac{1}{\sqrt{2\pi}\sigma_d} \exp\left(-\frac{(\alpha_d - \mu_d)^2}{2\sigma_d^2}\right) \end{cases} \quad (4.1)$$

where $\sigma_p, \mu_p, \sigma_d$ and μ_d are the standard deviation and the mean values of the distribution functions. The range of the coordinate variable α_p is 0 to 10, where 0 and 10 are corresponding to the base and tip-end position respectively; the value $\alpha_p=5$ denotes the lowest point of the crane-hook. The range of the direction variable α_d is also 0 to 10, where 0 and 10 are corresponding to leftward and rightward horizontal directions respectively; the value $\alpha_d=5$ denotes the downward normal direction.

The joint probability function of the two functions is defined as $g(\alpha_p, \alpha_d)$, that is expressed as

$$g(\alpha_p, \alpha_d) = \frac{1}{2\pi\sigma_p\sigma_d} \exp\left[-\frac{1}{2} \left\{ \frac{(\alpha_p - \mu_p)^2}{\sigma_p^2} + \frac{(\alpha_d - \mu_d)^2}{\sigma_d^2} \right\}\right] \quad (4.2)$$

where covariance between α_p and α_d is not considered in this time.

We discuss equation (4.2) as the continuous model. But the node information and the direction information is derived from LD database, so the probability density should be the discrete model. We round the continuous model into the discrete model. The interval of variable α_p and α_d is derived from the node number and the direction number of the LD database. The equation (4.3) is changed into discrete model as follows:

$$f(\alpha_p, \alpha_d) = \iint g(\alpha_p, \alpha_d) d\alpha_p d\alpha_d \quad (4.3)$$

4.2. PARAMETRIC ESTIMATION MODEL

The parameters $\sigma_p, \mu_p, \sigma_d$ and μ_d are estimated using Bayesian theory, regarding themselves as probabilistic variables. Estimation of such a parameter in Bayesian fashion[6] is given in the general form as:

$$f_{post}(p | \varepsilon) = \frac{P(\varepsilon | p) f_{pre}(p)}{\sum P(\varepsilon | p) f_{pre}(p)} \quad (4.4)$$

where p is the parameter to be estimated, f_{pre} is the prior probability density of p , f_{post} is the posterior probability density of p , ε is the observation data and P denotes the conditional probability.

In our study, we regard the probability density distribution as the joint probability density distribution because the distribution function in equation (4.2) has 4 parameters, or $\sigma_p, \mu_p, \sigma_d$ and μ_d . We can apply equation (4.4) as follows:

$$f_{post}(\sigma_p, \mu_p, \sigma_d, \mu_d | \varepsilon) = \frac{P(\varepsilon | \sigma_p, \mu_p, \sigma_d, \mu_d) f_{pre}(\sigma_p, \mu_p, \sigma_d, \mu_d)}{\sum P(\varepsilon | \sigma_p, \mu_p, \sigma_d, \mu_d) f_{pre}(\sigma_p, \mu_p, \sigma_d, \mu_d)} \quad (4.5)$$

where P can be expressed as:

$$P(\varepsilon | \sigma_p, \mu_p, \sigma_d, \mu_d) = \prod_{i=1}^{N_{data}} f(\alpha_p, \alpha_d) \quad (4.6)$$

where N_{data} is the number of sample data. We assume the range of the parameters as $0 \leq \mu_p, \mu_d \leq 10$ and $0 \leq \sigma_p, \sigma_d \leq 2$. The deviation values σ_p and σ_d are assume to be in smaller range because it is not desirable to set up a wide deviation in estimation, considering the cognition of the danger area of crane-hooks, which is our objective. The prior joint probability density distribution of their parameters is set with constant value (vague prior distribution).

We obtain parameters $\sigma_p, \mu_p, \sigma_d$ and μ_d in f_{post} by calculation of expression (4.5). In order to determine the probability density distribution at each of the parameters $\sigma_p, \mu_p, \sigma_d$ and μ_d , we define the marginal probability density functions. The distribution functions can be written as follows[7]:

$$f_{post,\sigma_p}(\sigma_p) = \sum_i \sum_j \sum_k f_{\sigma_p|\mu_p,\sigma_d,\mu_d}(\sigma_p | \mu_p, \sigma_d, \mu_d) f_{post,\mu_p}(\mu_p^i) f_{post,\sigma_d}(\sigma_d^j) f_{post,\mu_d}(\mu_d^k) \quad (4.7)$$

$$f_{post,\mu_p}(\mu_p) = \sum_l \sum_j \sum_k f_{\mu_p|\sigma_p,\sigma_d,\mu_d}(\mu_p | \sigma_p, \sigma_d, \mu_d) f_{post,\sigma_p}(\sigma_p^l) f_{post,\sigma_d}(\sigma_d^j) f_{post,\mu_d}(\mu_d^k) \quad (4.8)$$

$$f_{post,\sigma_d}(\sigma_d) = \sum_l \sum_i \sum_k f_{\sigma_d|\sigma_p,\mu_p,\mu_d}(\sigma_d | \sigma_p, \mu_p, \mu_d) f_{post,\sigma_p}(\sigma_p^l) f_{post,\mu_p}(\mu_p^i) f_{post,\mu_d}(\mu_d^k) \quad (4.9)$$

$$f_{post,\mu_d}(\mu_d) = \sum_l \sum_i \sum_j f_{\mu_d|\sigma_p,\mu_p,\sigma_d}(\mu_d | \sigma_p, \mu_p, \sigma_d) f_{post,\sigma_p}(\sigma_p^l) f_{post,\mu_p}(\mu_p^i) f_{post,\sigma_d}(\sigma_d^j) \quad (4.10)$$

where $f_{\sigma_p|\mu_p,\sigma_d,\mu_d}$, $f_{\mu_p|\sigma_p,\sigma_d,\mu_d}$, $f_{\sigma_d|\sigma_p,\mu_p,\mu_d}$ and $f_{\mu_d|\sigma_p,\mu_p,\sigma_d}$ are the posterior probability density of σ_p when μ_p, σ_d and μ_d are known, the posterior probability density of μ_p , the posterior probability density of σ_d and the posterior probability density of μ_d respectively. In this study, the estimation value of each parameter is the expectation value of the equations (4.7) to (4.10). The estimated parameter values can be expressed as:

$$\bar{\sigma}_p = \sum_l \sigma_p^l f_{post,\sigma_p}(\sigma_p^l) \quad (4.11)$$

$$\bar{\mu}_p = \sum_i \mu_p^i f_{post, \mu_p}(\mu_p^i) \quad (4.12)$$

$$\bar{\sigma}_d = \sum_j \sigma_d^j f_{post, \sigma_d}(\sigma_d^j) \quad (4.13)$$

$$\bar{u}_d = \sum_k \mu_d^k f_{post, \mu_d}(\mu_d^k) \quad (4.14)$$

4.3. IDENTIFICATION OF LOAD CONDITION BASED ON L-D DATABASE

The load condition of a damaged crane-hook is identified by means of the L-D database. The identification is performed according to the following minimization problem:

$$\text{Minimize } \tilde{\mathbf{x}} \mathbf{P} \tilde{\mathbf{x}}^T \quad \text{with respect to } \tilde{\mathbf{x}} \quad (4.15)$$

where $\tilde{\mathbf{x}} = [(\mathbf{x}_{ld}^{(1)} - \bar{\mathbf{x}}_f^{(1)}), (\mathbf{x}_{ld}^{(2)} - \bar{\mathbf{x}}_f^{(2)}), \dots, (\mathbf{x}_{ld}^{(N)} - \bar{\mathbf{x}}_f^{(N)}), (\alpha \mathbf{x}_{ld} - \alpha \bar{\mathbf{x}}_f)]$, $\mathbf{x}_{ld}^{(i)}$ denotes the difference between the deformed feature points obtained from the image of the damaged crane-hook and the corresponding calculated deformed nodal positions obtained from the L-D database. The matrix \mathbf{P} consists of weight coefficients and is expressed as

$$\mathbf{P} = \text{diag}[\alpha_1, \alpha_2, \dots, \alpha_N, \beta] \quad (4.16)$$

In the current study, the values are set as follows: $\alpha_1, \alpha_2, \dots, \alpha_N = 0.03$ and $\beta = 0.1$. The load condition in the L-D database corresponding to the solution of problem (4.15) is adopted for the image of the damaged crane-hook.

5. Estimated Damage Factor

The proposed damage factor estimation is conducted for the case of $N_{data} = 5, 10$ and 15 , where N_{data} is the number of data of damaged crane-hook. The results are shown in table 6.1. On the basis of the results, it is still not uncertain that the obtained distribution parameters are converged enough with the current number of data ($N_{data} = 15$); we can, however, confirm that there is a tendency towards the convergence.

If the true value exists, estimation value approaches the true one as the number of N_{data} increase. We expect the range including the true value with the different data numbers. We refer to the parameters of node information. The estimated the value of μ_p is in 6 to 8. It means that the center of the node parameter exists in more right position than the most downward position of the crane-hook. The estimated value of σ_p takes large numbers regardless the number of N_{data} . We refer to the parameters of direction

information. The estimated the value of μ_d is in 3 to 4.5. It means that the load direction is more left direction than the normal direction. σ_d decreases as the number of N_{data} increases.

We continue the consideration of each parameter. Roughly, when the number of N_{data} increases, the value of Bayesian estimation in standard deviation decrease. Therefore we estimate that the center value of true normal distribution of damage area is in 6 to 8. We find out that the possible force works on the more tip-end position than the most downward position in case of the crane-hooks are damaged. In the load direction, the standard deviation is large. This seems that the normal direction that is decided by the nodal position is deference from other normal direction. But when this result and the node information put together, we find out that the direction is toward under, or gravity direction because the center value is less than 5.

Considering mentioned above, the knowledge about the crane-hooks in this estimation is expressed with the figure 5.2 and figure 5.3.

Table 5.1 Estimated Parameters of Distribution of Load Condition				
Parameter	u_p	σ_p	u_d	σ_d
Range	$0 \leq u_p \leq 10$	$0 < \sigma_p \leq 2$	$0 \leq u_d \leq 10$	$0 < \sigma_d \leq 2$
Estimated value($N_{data}=5$)	6.78	1.63	4.28	1.80
Estimated value($N_{data}=10$)	7.81	1.35	3.22	1.72
Estimated value($N_{data}=15$)	7.25	1.46	3.54	1.66

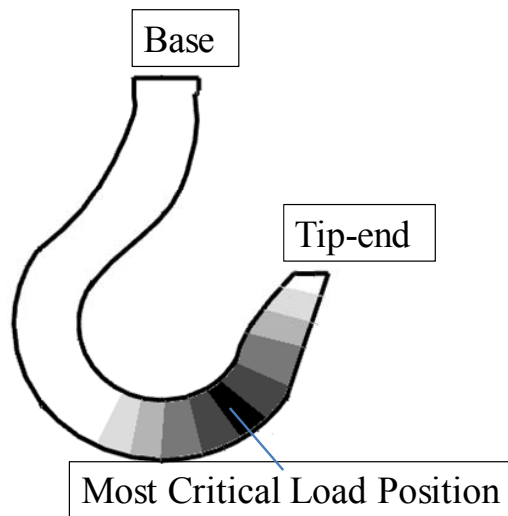


Figure 5.2 Estimated Critical Load Position

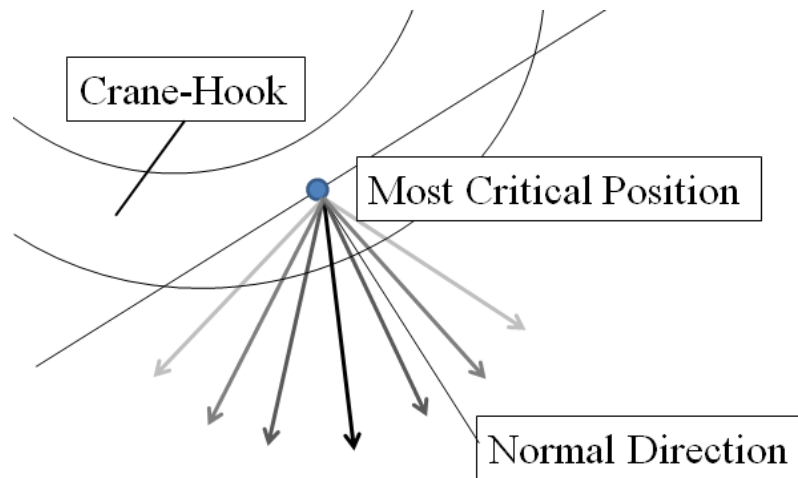


Fig 5.3 Estimated Critical Load Direction at Most Critical Load Position

Figure 5.2 and 5.3 shows the estimated distribution of position and direction of the critical load. The results indicate the followings on the critical load conditions:

- the load position is not at the lower center of the crane-hook
- the load direction is not downward normal.

On the basis of both of the above two observation of the results, load conditions that may not be appropriate can be applied on the crane-hooks from the mechanical point of view. More accurate and detailed evaluation of such conditions should be conducted.

6. Conclusion

We deal with the damage estimation of crane-hooks. The estimation is to find out the possible position of crane-hooks and the possible direction, which the force works on. First, load-deformation database that has the relation between the load condition of crane-hooks and its deformation using numerical calculation is constructed. Second, the feature points are detected from the crane-hook images. These are compared to the information of the LD database. The load condition that minimizes the error between the deformation of the LD database and the feature points that is obtained from the crane-hook image is used as the data in the damage estimation. The damage factor is estimated by using the data under the assumption that the distribution of the damage area is normal distribution. The result we obtained is that the load position lies between the most downward position and the tip-end position and the load direction is not downward normal in case of crane-hooks are damaged. In this time, we do not estimate the load magnitude because as the load magnitude is so large, the crane-hook is absolutely damaged. In terms of that meaning, we focus on the load position and the direction only. But this damage estimation is surely more detailed if the load

magnitude can be estimated because the load condition is completed by finding out the load magnitude. We create the probabilistic model including the load magnitude. This is our future work.

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