A Novel Robust Control Strategy for Interval Plants Using The Two Loop MFC and CDM

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Abstract. The important issues of robust control are that the controller must perform satisfactorily not just for one plant, but for a family of plants and the controller should ensure stability and performance despite the disturbances and modeling uncertainties. In this context, we propose a novel robust control method for interval plants that gives robustness against wide variations of plant parameters. The method is based on the combined merits of the two loop Model Following Control (MFC) and the Coefficient Diagram Method (CDM). The MFC structure offers two degree freedom of control and design flexibility to tackle tracking and disturbance rejection separately. CDM is one of the recently developed controller design methods using algebraic approach. In our proposed method, we modify the two loop control structure in accordance with the CDM. The controller is designed using CDM to satisfy the desired performance specifications. We establish mathematically the conditions that provide robustness to the system. We apply the proposed control strategy to two examples of interval plants from the existing literature and find the results to support the derived mathematical analysis. We also determine the parametric stability margins for both the examples using Bounded Phase Condition.

Keywords: MFC, CDM, Parametric Uncertainty, Robust Control, Bounded Phase Condition

1. Introduction

There always exists uncertainty between the system model and the actual plant. The main difficulties with the models of the physical systems are that the parameters determined may be less accurate due to the change in parameters with time, simplified assumptions made and existence of some unmodeled dynamics. The two loop control structure and the Coefficient Diagram method (CDM) are two such methods used to tackle the problem of robustness. The proposed method is a combination of the two, where in the two loop control structure is modified in accordance with the structure used in Coefficient Diagram method.

The two loop control structure (Model Following Control) is noted for the high robustness to plant perturbations, disturbance rejection and tracking properties (Stanislaw Skoczowski et al., 2003; Numsomran et al., 2006; Bhusnur and Ray, 2007). The two loop control structure (Åström and Wittenmark, 2006) consists of two unity feedback loops, one is the nominal plant loop and the other is the real plant loop. The model controller in the nominal plant loop is designed to produce

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the desired nominal plant output. This output acts as the reference input to the real plant loop. In (Skoczowski et al., 2005) a simple method to improve robustness of PID control is proposed using the two loop control structure. An effective fusion of relay feedback control with the two loop control structure has been presented in (Tsang and Li, 2005) to overcome deadzone nonlinearities. A robust control approach based on the adaptive model following concept as applied to induction servomotor drive has been discussed in (Yhanphairoje et al., 2004). Another paper (Osypiuk et al., 2006) deals with the application of two loop control structure to robotic manipulators. All of the above works involve PID controllers in the two loop control structure.

Coefficient Diagram method (CDM) is one of the recently developed controller design methods introduced by Shunji Manabe in 1991. It is an algebraic approach incorporating combined effects of classical control design and modern control design methodologies. In this method, characteristic polynomial and controller are simultaneously designed. A semilog diagram known as coefficient diagram (CD) is used as the main tool to analyze stability, speed of response and robustness features of a control system. A solution to ACC benchmark problem is presented in (Manabe, 1997) taking a good balance of stability, response and robustness. The controller design method is described in detail in (Manabe, 1998) including historical background, comparison with other control theories, mathematical relations and design procedure. The stability and instability conditions that form the basis of Coefficient Diagram method have been described in (Manabe, 1999). Some more basics of Coefficient Diagram method and recent developments have been dealt in (Manabe and Kim, 2000; Kim and Manabe, 2001; Hamamci and Koksal, 2001; Hamamci et al., 2001; Kim et al., 2). The effectiveness of the Coefficient Diagram as a design tool is focused in (Manabe, 2003; Manabe, 2005). An improved and simplified literature for CDM is presented in (Koksal and Hamamci, 2004) including an introduction to computer aided design of CDM controllers.

In this paper we propose a novel robust control methodology by modifying each loop of the two loop control structure in the form of CDM control structure. We obtain the mathematical conditions that contribute to the robustness of the modified two loop structure. We apply the proposed methodology to the two examples of interval plants to examine the effectiveness of the proposed control strategy. Rest of the paper is organized as follows: In the next two sections we brief the basics of the two loop control structure and CDM. In Section 4 we describe the modified two loop structure, propose the robust control strategy, brief the Bounded Phase Condition of robust control and give the input output relationship between disturbance input and the actual plant output. In Section 5 we discuss the application of the proposed methodology to the two examples of interval plants and relevant simulation results. Section 6 is the concluding part.

2. Basics of the Two Loop Control Structure

In the two loop control structure, as shown in Figure 1, $R(s)$ is reference input and $M(s)$ is the nominal model to represent the dynamics of the linear part of the plant. $C_m(s)$ is model controller that ensures a desired tracking performance such that the plant generates satisfactory reference output $Y_m(s)$ and $Z(s)$ is the disturbance input. The second loop is formed using $C(s)$ as a robust controller and the actual plant $P(s)$. The transfer function of the actual plant can be expressed as

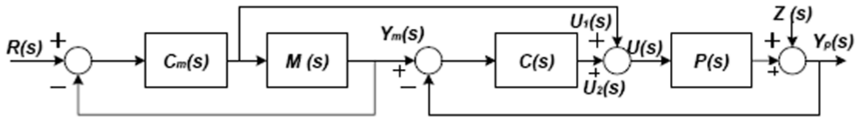

Figure 1. Two loop control structure.

(Stefani et al., 1995)

$$P(s) = M(s)(1 + \triangle(s)) \tag{1}$$

where $\triangle(s)$ is multiplicative perturbation. The plant $P(s)$ is controlled by the signals $U_1(s)$, which also controls the nominal plant and $U_2(s)$, obtained from robust controller $C(s)$. This provides a better control signal to the plant $P(s)$, so that its output can track the nominal model output $Y_m(s)$ in the presence of modeling errors. The plant output $Y_1(s)$, considering only reference input is given by (Bhusnur and Ray, 2007)

$$Y_1(s) = \frac{C_m(s)(1 + C(s)M(s))P(s)}{(1 + C_m(s)M(s))(1 + C(s)P(s))}R(s) \tag{2}$$

and the plant output $Y_2(s)$, considering only disturbance input is given by

$$Y_2(s) = \frac{1}{(1 + C(s)P(s))}Z(s) \tag{3}$$

Then, the real plant output $Y_p(s)$ obtained from (2) and (3) is

$$\begin{aligned} Y_p(s) \;=\; & \frac{(s)(1 + C(s)M(s))P(s)}{(1 + C_m(s)M(s))(1 + C(s)P(s))}\frac{(1 + C_m(s)M(s))}{C_m(s)M(s)}Y_m(s) \\ & + \frac{1}{(1 + C(s)P(s))}Z(s) \end{aligned} \tag{4}$$

When $C_m(s) \neq C(s)$, simplifying (4) we get (Bhusnur and Ray, 2007)

$$Y_p(s) = \left[1 + \frac{\triangle(s)}{(1 + C(s)M(s)(1 + \triangle(s)))}\right]Y_m(s) + \frac{1}{(1 + C(s)\;M(s)(1 + \triangle(s)))}Z(s) \tag{5}$$

From (5) it is seen that with $C_m(s)$ designed for good tracking quality, $C(s)$ can be used to reduce the effects of parameter variations and disturbances if $|C(s)M(s)| \gg 1$ (Skoczowski et al., 2005). Also, if there are no modeling errors, that is $\triangle(s) = 0$, this control structure will force the plant output $Y_p(s)$ to follow the nominal model output $Y_m(s)$. The two degree freedom of control offered by this structure is the reason for the design flexibility. The best feature about the structure is that the controllers of the two loops can be designed independently using any controller design methodology.

3. Basics of Coefficient Diagram Method

3.1. Block Diagram and Stability Conditions

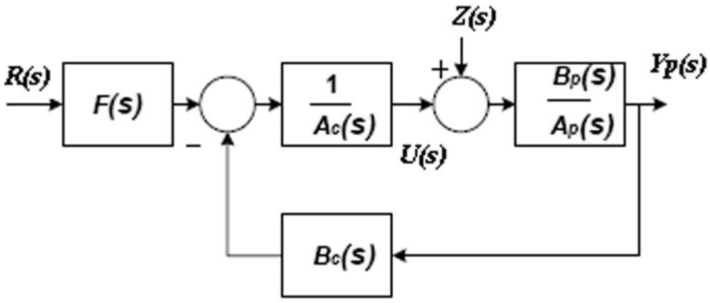

Figure 2. Block Diagram of CDM.

The standard block diagram for CDM (Manabe, 1997) is shown in Figure 2. $B_p(s)$ and $A_p(s)$ are respectively the numerator and denominator polynomials of the plant . $B_c(s)$ is the feedback numerator polynomials of the controller transfer function while $A_c(s)$ is the forward denominator polynomial of the same. $B_c(s)$ and $A_c(s)$ are designed to meet the desired transient response and the prefilter $F(s)$ is used to provide the steady state gain. The output of the closed loop system is

$$Y_p\left(s\right) = \frac{B_p(s)F(s)}{A_{cl}(s)}R(s) + \frac{A_c(s)B_p(s)}{A_{cl}(s)}Z(s) \tag{6}$$

where $A_{cl}(s)$ is the characteristic polynomial and is given by

$$A_{cl}(s) = A_p(s)A_c(s) + B_p(s)B_c(s) = a_n s^n + \cdots + a_1 s + a_0 = \sum_{i=0}^{n} a_i s^i \tag{7}$$

where a_is are the coefficients. The design parameters of CDM are the equivalent time constant τ, the stability index γ_i, and stability limit γ_i^* and are respectively defined as

$$\tau = \frac{a_1}{a_0} \tag{8}$$

$$\gamma_i = \frac{a_i^2}{a_{i+1}\ a_{i-1}}\ , i = 1, 2, \ldots\ , (n-1), \gamma_n = \gamma_0 = \infty \tag{9}$$

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}}\ , i = 1, 2, \cdots, (n-1) \tag{10}$$

and

$$a_i = \frac{a_0 \tau^i}{\gamma_{i-1}\gamma_{i-2}^2 \cdots \gamma_1^{i-1}} \tag{11}$$

Hence,

$$a_i = \left[\ a_n \ \ a_{n-1} \ \ . \ \ . \ \ a_1 \ \ a_0\ \right] \tag{12}$$

$$\gamma_i = [\ \gamma_{n-1}\ \ .\ \ .\ \ .\ \ \gamma_1\ ] \tag{13}$$

$$\gamma_i^* = [\ \gamma_{n-1}^*\ \ .\ \ .\ \ .\ \ \gamma_1^*\ ] \tag{14}$$

The equivalent time constant specifies the speed of time response, the stability indices and limits specify stability and nature of time response; and the variation of stability indices due to plant perturbations indicates the robustness. The characteristic equation can be expressed as (Manabe, 1998; Kim and Manabe, 2001; Koksal and Hamamci, 2004)

$$A_{cl}(s) = a_0 \left[\left\{ \sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^{j}} \right) (\tau s)^i \right\} + \tau s + 1 \right] \tag{15}$$

The Stability conditions used in CDM are as follows (Manabe, 1999; Manabe and Kim, 2000);

1. On the basis of Routh-Hurwitz Criterion, the stability condition for third and fourth order systems is

$$\gamma_i\ > \gamma_i^*\ , i = 1 \sim (n-1) \tag{16}$$

2. The sufficient condition for stability for the systems of degree five and above is obtained by Lipatov (Manabe, 1999) given by

$$\gamma_i\ > 1.12\ \gamma_i^*, \forall\ i = 2 \sim (n-2) \tag{17}$$

3. The sufficient condition for instability is given by

$$(\gamma_{i+1}\gamma_i) \leq 1, for\ some\ i = 1 \sim (n-2) \tag{18}$$

3.2. Coefficient Diagram and Controller Design

The Coefficient Diagram (CD) is a semi-log diagram of the coefficients a_i of the polynomial in logarithmic scale on lefthand side vertical axis and the corresponding power of s in linear scale on horizontal axis, also the stability index γ_i, stability limit γ_i^* and the equivalent time constant τ are read on righthand side vertical axis (Manabe, 1999; Manabe and Kim, 2000). τ is expressed by a line connecting 1 to τ. If the curvature of the coefficient curve is large the system is more stable on account of larger stability indices. If the curve is left end down it indicates that τ is small and the response is fast. As the coefficients of the characteristic polynomial are related to the parameters of the plant and controller (7) the designer can visually assess the sensitivity and robustness features by observing the deformation of the coefficient curve due to the parameter variations. The controller design using CDM is summarized in the following paragraph;

Let the degree of the denominator polynomial of the transfer function be n_c and that of the numerator polynomial be m_c. Taking into account the effect of disturbance if any, let the controller polynomials be given as (Hamamci and Koksal, 2001; Koksal and Hamamci, 2004)

$$A_c(s) = \sum_{i=0}^{n_c} l_i s^i\ , B_c(s) = \sum_{i=0}^{m_c} k_i s^i \tag{19}$$

The prefilter $F(s)$ is chosen as (Koksal and Hamamci, 2004)

$$F(s) = \left(\frac{A_{cl}(s)}{B_p(s)} \right)_{s=0} \tag{20}$$

According to the specified settling time t_s requirement τ is chosen as (Manabe, 1999)

$$\tau = \frac{t_s}{(2.5 \sim 3)} \tag{21}$$

The recommended values of the stability indices are (Manabe, 1997)

$$[\, \gamma_{n-1} = \gamma_{n-2} = \ . \ . \ . = \gamma_2 = 2, \ \gamma_1 = 2.5 \,] \tag{22}$$

and the stability limits are found using (10). Thus, from (15) the target characteristic polynomial is obtained. Further making use of (7) and (19) the resulting equation with plant parameters and controller coefficients is compared with the target polynomial and the value of the controller coefficients is obtained. Finally, Coefficient Diagram (CD) is drawn for the coefficients of this characteristic polynomial. The necessity of any modification can be easily determined from CD and the process may be repeated with modified set of design parameters.

4. Proposed Control Structure

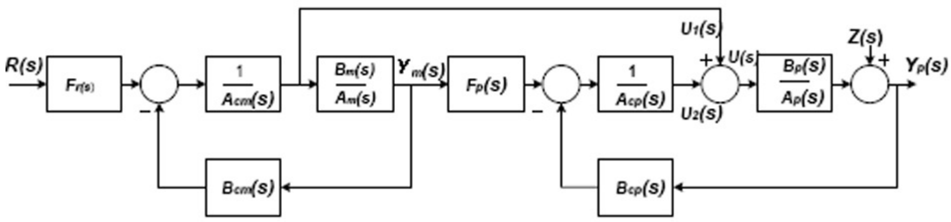

Figure 3. Control structure of the proposed method.

In the proposed method the block diagram representation of CDM is used in each loop of the modified two loop structure. In the first loop $B_m(s)$ and $A_m(s)$ are the numerator and denominator polynomials of the transfer function of the *reference plant* $M(s)$ respectively. The reference plant is chosen by *modifying suitably* the known nominal model of the actual plant. $F_r(s)$ and $B_{cm}(s)$ are the reference numerator and the feedback numerator polynomials while $A_{cm}(s)$ is the forward denominator polynomial of the controller transfer function. In the second loop $B_p(s)$ and $A_p(s)$ are respectively the numerator and denominator polynomials of the actual plant $P(s)$ with uncertainties. The second loop is similarly represented in CDM form.

4.1. Controller Design

The block diagram of the proposed control structure can be reduced and the transfer function between $Y_m(s)$ and $Y_p(s)$ is obtained as

$$\frac{Y_p(s)}{Y_m(s)} = \frac{B_p(s)}{B_m(s)} \left(\frac{B_m(s)F_p(s) + A_m(s)A_{cp}(s)}{A_{cp}(s)A_p(s) + B_{cp}(s)B_p(s)} \right) \tag{23}$$

Since the actual output should track the reference model output

$$\left(\frac{\Delta(s)}{1 + \Delta(s)} \right) A_{cp}(s) = M(s) \left(B_{cp}(s) - F_p(s) \right) \tag{24}$$

If

$$\frac{\Delta(s)}{1 + \Delta(s)} \simeq 1 \tag{25}$$

then,

$$A_{cp}(s) \simeq M(s) \left(B_{cp}(s) - F_p(s) \right) \tag{26}$$

Here, we choose $M(s)$ such that it satisfies (25). This can be conveniently done once the nominal model of the plant to be controlled is known. The controller polynomials $B_{cm}(s)$ and $A_{cm}(s)$ for this plant are designed using CDM. Thus, even if the parameters of the actual plant $P(s)$ deviate from the nominal plant parameter values, (25) still holds good. Further, the controller designed using (26) gives robust performance for wide range of parameter variations in the actual plant around the nominal values.

In this paper the proposed control strategy is explained using type 1 position control system. Let the nominal model of the plant be given by

$$P_{nom}(s) = \frac{K_m}{sA'_p(s)} \tag{27}$$

where K_m is the gain of the position control system, $A'_p(s)$ is the rest of the denominator polynomial with poles other than s, $M(s)$ is chosen as reference plant to satisfy (25). Let $M(s)$ be expressed as

$$M(s) = \frac{xK_m}{sA'_p(s)} = \frac{B_m(s)}{A_m(s)} \tag{28}$$

where x is some integer so as to satisfy (25). In the modified two loop control structure the loop with $M(s)$ is devoid of any disturbance, hence the controller polynomials $B_{cm}(s)$ and $A_{cm}(s)$ are chosen to be of order $n-1$ and are designed using CDM (Koksal and Hamamci, 2004). Using (19)

$$\frac{B_{cm}(s)}{A_{cm}(s)} = \frac{k_{n-1}s^{n-1} + k_{n-2}s^{n-2} + \cdots + k_1 s + k_0}{l_{n-1}s^{n-1} + l_{n-2}s^{n-2} + \cdots + l_1 s + l_0} \tag{29}$$

Therefore, characteristic equation considering the loop with the reference plant is given by

$$A_{cl}(s) = (l_{n-1}s^{n-1} + l_{n-2}s^{n-2} + \cdots + l_1 s + l_0)sA'_p(s) + xK_m(k_{n-1}s^{n-1} + k_{n-2}s^{n-2} + \cdots + k_1 s + k_0) \tag{30}$$

From (20)

$$F_r(s) = k_0 \tag{31}$$

The steady state value of $Y_p(s)$ for unit step input can be found using (23), and is given by

$$\lim_{s\to 0} Y_p(s) = \lim_{s\to 0} \frac{F_p(s)}{B_{cp}(s)} Y_m(s) \tag{32}$$

Therefore, for steady state tracking it is required that

$$\lim_{s\to 0} \frac{F_p(s)}{B_{cp}(s)} = 1 \tag{33}$$

To meet the constraint given by (33), from (29) and (31), we choose

$$B_{cp}(s) = k_1' s + k_0' = B_{cm}(s) \tag{34}$$

Where k_1' and k_0' are constants and

$$F_p(s) = F_r(s), \ i.e.\ k_1' = k_1, \ k_0' = k_0 \tag{35}$$

Thus using (26), (34) and (35) we design the real plant controllers. Necessary modifications in the value of controller parameters can be easily visualised using Coefficient Diagram.

4.2. Stability Analysis

We apply the Bounded Phase Condition of the parametric approach of robust control (Bhattacharyya et al., 1995) to examine whether the controllers robustly stabilize the plant.

Let the closed loop characteristic polynomial of the real plant loop be expressed as

$$\delta(s,p) = a_1(s)p_1 + a_2(s)p_2 + \cdots + a_l(s)p_l + a_0(s) \tag{36}$$

where p_i is a component of real parameter vector

$$\mathbf{p} := [p_1, p_2, p_3 \cdots\cdots p_l]^T$$

and $a_i(s)$ for $i = 0, 1, 2, \cdots l$ are fixed polynomials. Also

$$\delta(s,\mathbf{p}) = \delta_0(\mathbf{p}) + \delta_1(\mathbf{p})s + \delta_2(\mathbf{p})s^2 + \cdots + \delta_n(\mathbf{p})s^n$$

where each component δ_i is linear function of $\mathbf{p}$. We assume that $\mathbf{p}$ lies in an uncertainty set which is box-like:

$$\mathbf{\Pi} := \left\{\mathbf{p} : p_i^- \leq p_i \leq p_i^+ i = 1, 2, \cdots l\right\} \tag{37}$$

Thus the set of characteristic polynomials

$$\mathbf{\Delta}(s) := \{\delta(s,\mathbf{p}) : \mathbf{p} \in \mathbf{\Pi}\}$$

The vertices $\mathbf{V}$ of $\mathbf{\Pi}$ are obtained by setting each p_i to p_i^+or p_i^-

$$\mathbf{V} := \left\{\mathbf{p} : p_i = p_i^- \ or \ p_i = p_i^+, i = 1, 2, \cdots l\right\}$$

and the vertex polynomials are

$$\mathbf{\Delta}_v(s) := \{\delta(s, \mathbf{p}) : \mathbf{p} \in \mathbf{V}\} \tag{38}$$

For a convex polygon $\mathcal{P}$ in the complex plane with vertices $\mathbf{V} := [v_1, v_2, \cdots]$, let

$$\phi_{v_i} = \arg\left(\frac{v_i}{p_0}\right) \tag{39}$$

where p_0 is any arbitrary point in $\mathcal{P}$. The angle subtended at the origin by the convex polygon is given by

$$\phi = \phi^+ - \phi^- \tag{40}$$

where

$$\phi^+ = \sup_{v_i \in \mathbf{V}} \phi_{v_i}, 0 \leq \phi^+ \leq \pi, \phi^+ = \sup_{v_i \in \mathbf{V}} \phi_{v_i}, 0 \leq \phi^+ \leq \pi \tag{41}$$

In the complex plane $\mathcal{C}$ let the stability region $\mathcal{S} \in \mathcal{C}$ be the open left half plane and its boundary be denoted as $\partial\mathcal{S}$, then we have the following Theorem

THEOREM 4.1. (Bounded Phase Theorem) *(*Bhattacharyya et al., 1995*) Under the assumptions*

1. *every polynomial in $\mathbf{\Delta}(s)$ is of the same degree ($\delta_n(\mathbf{p}) \neq 0, \mathbf{p} \in \mathbf{\Pi}$),*
2. *at least one polynomial in $\mathbf{\Delta}(s)$ is stable with respect to $\mathcal{S}$,*

the set of polynomials $\mathbf{\Delta}(s)$ is stable with respect to the open stability region $\mathcal{S}$ if and only if

$$\phi_{\mathbf{\Delta}_v}(s^*) < \pi, \forall s^* \in \partial\mathcal{S} \tag{42}$$

The proof of the theorem is given in (Bhattacharyya et al., 1995). Computationally, we need to evaluate the maximum phase difference across the vertices of $\mathbf{\Pi}$.

Using the bounded phase condition (42) we examine robust stability and also find the parametric stability margin i.e., the maximal range of parameter $\mathbf{p}$ in the uncertainty set $\Omega(\varepsilon)$ about the nominal value $\mathbf{p}^0$, for which the closed loop stability is preserved. The parametric uncertainty set is defined by

$$\Omega(\varepsilon) := \left\{\mathbf{p} : \left\|\mathbf{p} - \mathbf{p}^0\right\| \leq \varepsilon\right\}$$

In the two loop control structure the stability of the whole system relies on the stability of the loop with the real plant since the nominal plant loop is certainly stabilized (Tsang and Li, 2005).

4.3. Disturbance Rejection

From (3) the input-output relationship between the disturbance input and the real plant output considering $R(s) = 0$ is given by

$$\frac{Y_p(s)}{Z(s)} = \frac{A_p(s)A_{cp}(s)}{A_p(s)A_{cp}(s) + B_p(s)B_{cp}(s)} \tag{43}$$

using which the disturbance rejection property can be analyzed. For a type 1 plant and the proposed controller structure the steady state value of the output to the step disturbance input is zero.

5. Design Examples

We consider two examples of type 1 system from the existing literature to illustrate the proposed methodology. The simulation results are obtained using MATLAB and SIMULINK environment.

5.1. A Second Order Type one System

We consider a second order type one position control system taken from (Tsang and Li, 2005) as the real plant

$$P(s) = \frac{K_m}{s(T_m s + 1)},\ K_m \in [12, 25], T_m \in [0.05\ , 0.5] \tag{44}$$

5.1.1. *Controller Design*

Let the nominal plant with the settling time specification as $t_s \leq 1.5\ sec$ be

$$P_{nom}(s) = \frac{K_m}{sA'_p(s)} = \frac{18.3}{s(0.1s + 1)} \tag{45}$$

Using (25) we choose the reference plant

$$M(s) = \frac{1}{s(0.1s + 1)} \tag{46}$$

Following (29) the controller polynomials for this reference plant are

$$\frac{B_{cm}(s)}{A_{cm}(s)} = \frac{k_1 s + k_0}{l_1 s + l_0} \tag{47}$$

The design of the controller parameters is done using standard Manabe Form of CDM with

$$\tau = 0.4\ sec \tag{48}$$

The stability limits are

$$[\ \gamma_2\ \ \gamma_1\ ] = [\ 2\ \ 2.5\ ] \tag{49}$$

Now, letting a_0 arbitrarily equal to 1 the target characteristic equation of the reference plant loop can be obtained using (15) as

$$A_{cl}(s) = 5.12 \times 10^{-3} s^3 + 0.064 s^2 + 0.4 s + 1 \tag{50}$$

Also from (30)

$$A_{cl}(s) = 0.1 l_1 s^3 + (l_1 + 0.1 l_0) s^2 + (l_0 + k_1) s + k_0 \tag{51}$$

Comparing (50) and (51) we obtain the controller parameters for the reference plant. Therefore,

$$\frac{B_{cm}(s)}{A_{cm}(s)} = \frac{0.2720 s + 1}{0.0512 s + 0.128} \tag{52}$$

Further from (31)

$$F_r(s) = 1 \tag{53}$$

Once the reference plant loop is designed, the controllers for the actual plant can be derived from (34) and (35) , the feedback controller is obtained as

$$B_{cp}(s) = 0.272 s + 1 \tag{54}$$

and the cascade controller is obtained as

$$\frac{1}{A_{cp}(s)} = 0.36764 s + 3.6764 \tag{55}$$

Both the controllers can be realized by using PD controllers.

5.1.2. *Simulation Results*

The simulation results are discussed with respect to Performance Analysis, Effect of Disturbance and Robust stability.

5.1.2.1. *Performance Analysis.* Figure 4 shows the reference plant output generated by the controller designed using CDM and the simulated step responses of the perturbed plant taking the extreme values in T_m and K_m as given by (44). The reference plant output is the input to the loop with the real plant. Figure 5 shows the Coefficient Diagrams taking the extreme values in T_m and K_m as given by (44). From Figure 4 we observe that the real plant output tracks the reference plant output even in the presence of large plant parameter variations and similar result is depicted in Figure 5. We observe very little deformation in the Coefficient Diagrams in the presence of large plant parameter variations.

5.1.2.2. *Effect of Disturbance.* Figure 6 shows the effect of a unit step disturbance input on the interval plant. We observe that the design offers disturbance rejection to step disturbance inputs.

5.1.2.3. *Robust Stability.* First we apply Bounded Phase Condition to examine whether the controllers robustly stabilize the plant. Next, we also determine the largest possible excursion of

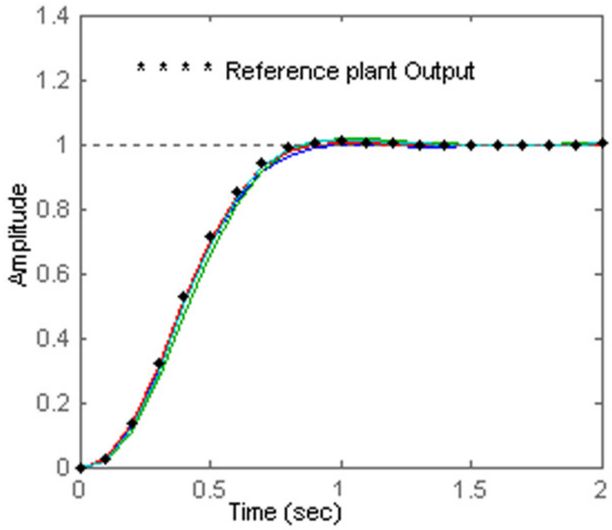

Figure 4. Step responses of the interval plants of Example 1.

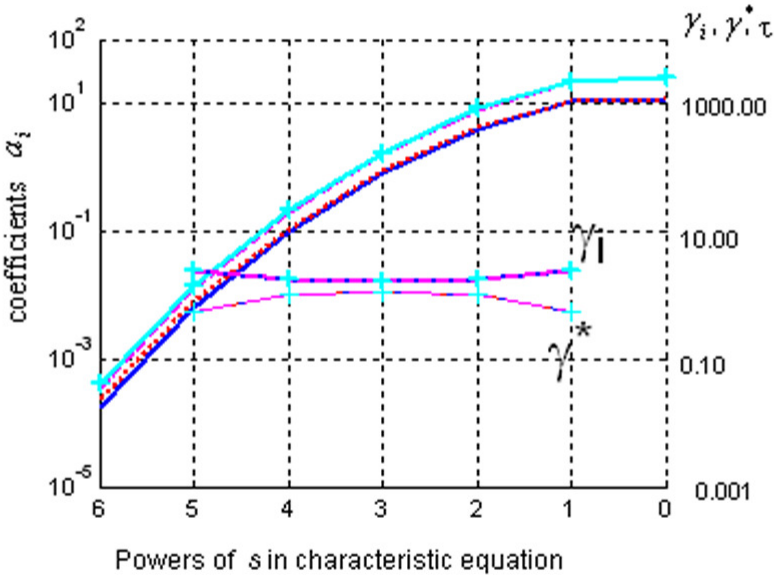

Figure 5. Coefficient Diagrams for the interval plants of Example 1.

parameters ε that preserves the closed loop stability. The characteristic equation for the loop with the real plant is

$$\delta(s) = K_m a_1(s) + T_m a_2(s) + a_0(s) \tag{56}$$

where

$$a_1(s) = 0.0272s^2 + 0.372s + 1, a_2(s) = 0.272s^2, a_0(s) = 0.272s \tag{57}$$

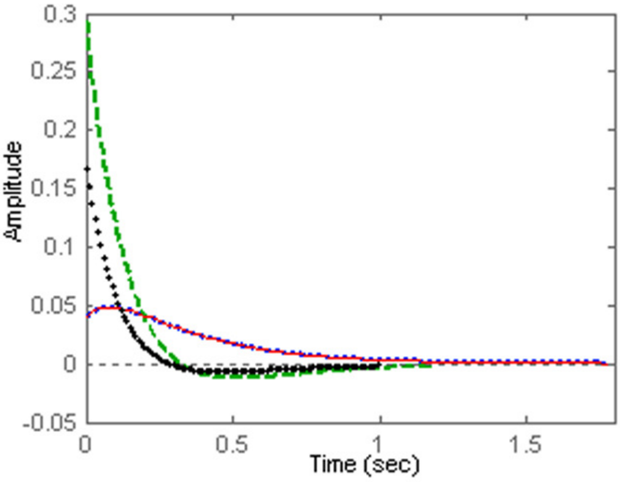

Figure 6. Responses to a unit step disturbance input for the interval plants of Example 1.

The family of polynomials whose stability is examined is expressed as

$$\mathbf{\Delta}(s) = \{\delta(s) : K_m \in [12 - \varepsilon, 25 + \varepsilon], T_m \in [0.05 - \varepsilon, 0.5 + \varepsilon]\} \quad (58)$$

At $\omega = 0$, $\varepsilon = 0$ we have

$$\mathbf{\Delta}(j0) = K_m = [12, 25] \quad (59)$$

Bounded Phase Condition can be applied since $\mathbf{\Delta}(s)$ has at least one stable polynomial, i.e., $0 \notin \mathbf{\Delta}(j\omega)$ for at least one value of ω. The maximum phase difference over all the vertices with respect to origin of the complex plane is shown by the curve with $\varepsilon = 0$ in the Figure 7. The maximum phase difference is around 11 degree (less than180 degree) and indicates that the controllers given by (54) and (55) robustly stabilize $\mathbf{\Delta}(s)$ for the uncertainty interval given in (44).

Now, we apply the Bounded Phase Condition with increasing values of ε until maximum phase difference reaches 180 degree. Figure 7 shows that the plot of maximum phase difference over all vertices against the frequency ω approaches 180 degree for $\varepsilon = 1.2$. Thus, parameter perturbation can be increased to a value around 1.2 till the system becomes unstable.

5.2. A Third Order Type one System

We consider a third order type one control system taken from (Hamamci and Koksal, 2001) as the real plant

$$P(s) = \frac{K_m}{s(T_m s + 1)(T_f s + 1)}, K_m \in [0.5, 1.5], T_m = [0.5, 2], T_f = [0.1, 0.5] \quad (60)$$

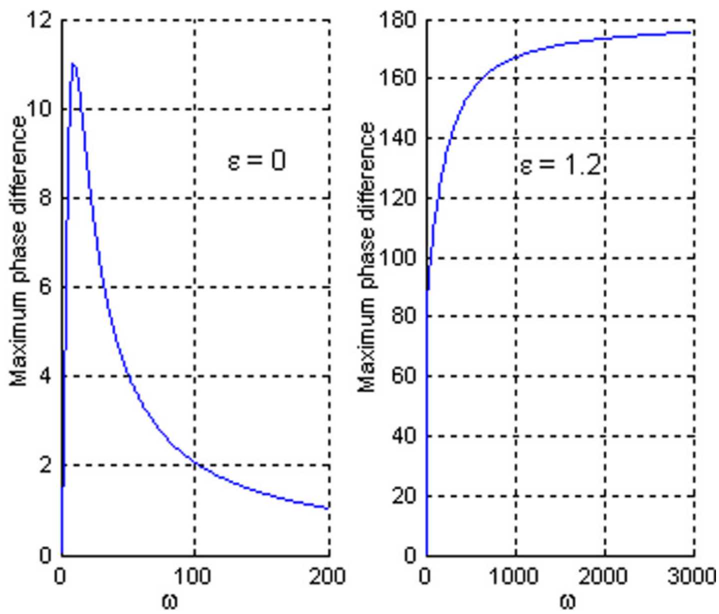

Figure 7. Maximum Phase difference of the vertex polynomials of Example 1, for $\varepsilon = 0$ and $\varepsilon = 1.2$.

5.2.1. *Controller Design*

Let the nominal plant with performance specifications $t_s \leq 8$ *sec* and overshoot $< 10\%$ be

$$P_{nom}(s) = \frac{K_m}{sA'_p(s)} = \frac{1}{s(0.25s+1)(s+1)} \tag{61}$$

Using (25) we choose the reference plant

$$M(s) = \frac{0.1}{s(0.25s+1)(s+1)} \tag{62}$$

Following (29) the controller polynomials for this reference plant are

$$\frac{B_{cm}(s)}{A_{cm}(s)} = \frac{k_2 s^2 + k_1 s + k_0}{l_2 s^2 + l_1 s + l_0} \tag{63}$$

The design of the controller parameters is done using CDM with

$$\tau = 2.0 \; sec \tag{64}$$

The stability limits are

$$[\, \gamma_4 \;\; \gamma_3 \;\; \gamma_2 \;\; \gamma_1 \,] = [\, 4 \;\; 3 \;\; 3 \;\; 2 \,] \tag{65}$$

The target characteristic equation of the reference plant loop can be obtained using (15) as

$$A_{cl}(s) = 2.1 \times 10^{-3} s^5 + 0.0741 s^4 + 0.6667 s^3 + 2s^2 + 2s + 1 \tag{66}$$

Also,

$$A_{cl}(s) = 0.25l_2 s^5 + (1.25l_2 + 0.25l_1)\, s^4 + (l_2 + 1.25l_1 + 0.25l_0)\, s^3 + (l_1 + 1.25l_0 + 0.1k_2)\, s^2 + (l_0 + 0.1k_1)\, s + 0.1k_0 \tag{67}$$

Comparing (66) and (67) we obtain the controller parameters for the reference plant. Therefore,

$$\frac{B_{cm}(s)}{A_{cm}(s)} = \frac{0.4733s^2 + 6.4198s + 10}{0.0082s^2 + 0.2551s + 1.358} \tag{68}$$

$$F_r(s) = 10 \tag{69}$$

Let

$$B_{cp} = k_2' s^2 + k_1' s + k_0' \tag{70}$$

Where k_2', k_1' and k_0' are constants.

$$k_1' = k_1 \tag{71}$$

and

$$k_0' = k_0 \tag{72}$$

It is investigated mathematically using (43) that reducing the value of k_2' improves disturbance rejection and reduces the corresponding settling time without disturbing the curvature of the coefficient curve. Therefore the value of k_2' is not taken equal to k_2 but reduced suitably to 0.1. Thus

$$B_{cp} = 0.1s^2 + 6.4198s + 10 \tag{73}$$

From (26)

$$\frac{1}{A_{cp}} = \frac{(2.5s + 10)(s + 1)}{0.1s + 6.4198} \tag{74}$$

The controllers can be realized using PD controllers and suitable derivative filters.

5.2.2. *Simulation Results*

The simulation results are discussed with respect to Performance Analysis, Effect of Disturbance and Robust stability.

5.2.2.1. *Performance Analysis.* Figure 8 shows the reference plant output generated by the controller designed using CDM and the simulated step responses of the perturbed plant taking the extreme values in K_m, T_m and T_f as given by (60). Figure 9 shows the Coefficient Diagrams taking the extreme values in K_m, T_m and T_f as given by (60). From Figure 8 we observe that the real plant output tracks the reference plant output even in the presence of large plant parameter variations and similar result is depicted in Figure 9. Less deformation in the Coefficient Diagrams in the presence of large plant parameter variations indicates robustness.

5.2.2.2. *Effect of Disturbance.* Figure 10 shows the effect of a unit step disturbance input on the interval plant of Example 2. From Figure 10 we observe that the design offers disturbance rejection

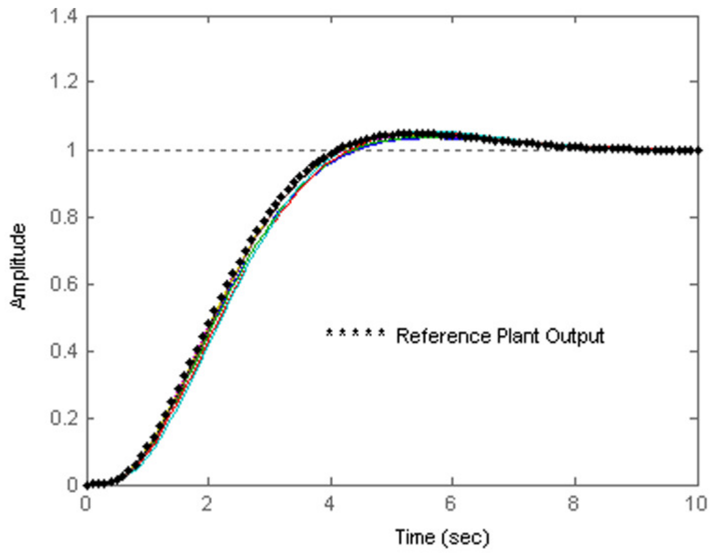

Figure 8. Step responses of the interval plants of Example 2.

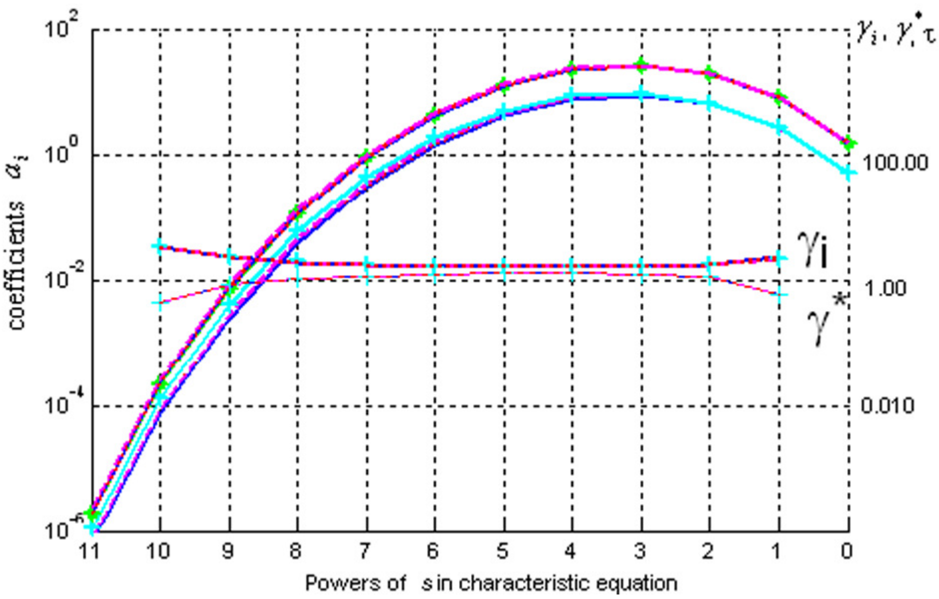

Figure 9. Coefficient Diagrams for the interval plants of Example 2.

to step disturbance inputs.

5.2.2.3. *Robust Stability.* In this Section, first we apply Bounded Phase Condition to examine whether the controllers robustly stabilize the plant. we also determine the largest possible excursion of parameters ε that preserves the closed loop stability. The characteristic equation for the loop

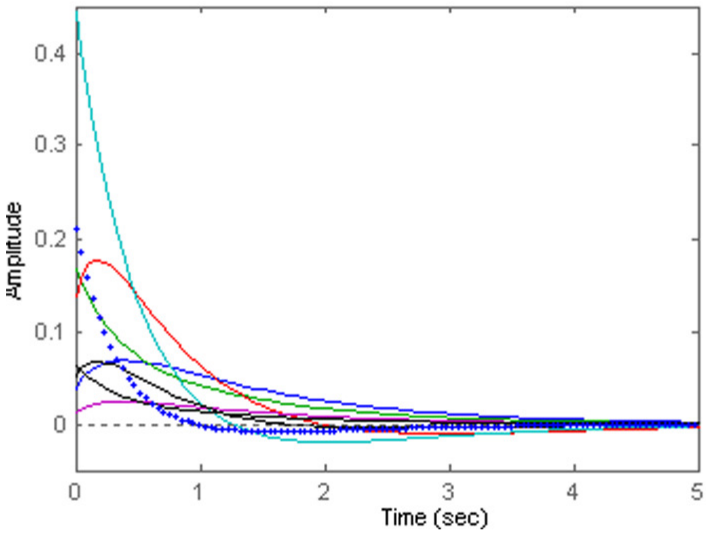

Figure 10. Responses to a unit step disturbance input for the interval plants of Example 2.

with the real plant is obtained as

$$\begin{aligned}\delta(s) \;=\; & K_m(0.025s^4 + 1.73s^3 + 10.625s^2 + 18.92s + 10) \\ & + T_m(0.01s^3 + 0.642s^2) \\ & + T_f(0.01s^3 + 0.642s^2) \\ & + \alpha(0.01s^4 + 0.642s^3) \\ & + 0.01s^2 + 0.642s\end{aligned}$$

where

$$\alpha = T_m T_f \tag{75}$$

The family of polynomials whose stability is examined is expressed as

$$\mathbf{\Delta}(s) = \left\{ \begin{array}{c} \delta(s) : K_m \in [0.5-\varepsilon, 1.5+\varepsilon], T_m \in [0.5-\varepsilon, 2+\varepsilon] \\ T_f = [0.1-\varepsilon, 0.5+\varepsilon], \alpha = [0.05-\varepsilon, 1+\varepsilon] \end{array} \right\} \tag{76}$$

At $\omega = 0$, $\varepsilon = 0$ we have

$$\mathbf{\Delta}(j0) = 10, \; K_m = [5, 15] \tag{77}$$

Bounded Phase Condition can be applied since $\mathbf{\Delta}(s)$ has at least one stable polynomial, i.e., $0 \notin \mathbf{\Delta}(j\omega)$ for at least one value of ω. The maximum phase difference over all the vertices with respect to origin of the complex plane is shown by the curve with $\varepsilon = 0$ in the Figure 11. The maximum phase difference is around 25 degree (less than180 degree) and indicates that the controllers given by (73) and (74) robustly stabilize $\mathbf{\Delta}(s)$ for the uncertainty interval given in (60).

We apply the Bounded Phase Condition with increasing values of ε until maximum phase difference reaches 180 degree. Figure 11 shows that the plot of maximum phase difference over all

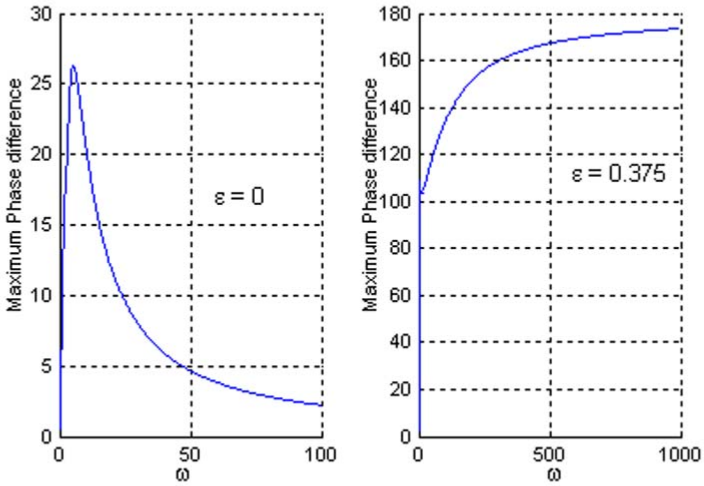

Figure 11. Maximum Phase difference of the vertex polynomials of Example 2, for $\varepsilon = 0$ and $\varepsilon = 0.375$.

vertices against the frequency ω approaches 180 degree for $\varepsilon = 0.375$. Thus, parameter perturbation can be increased to a value around 0.375 till the system becomes unstable.

6. Conclusion

In this paper a novel control strategy is proposed by modifying the two loop control structure according to the CDM control structure. The proposed method has been explained taking two interval plants of type 1 system.

In each case the controller design, the effect of disturbance and stability analysis have been dealt separately. The mathematical conditions that contribute to the robustness have been derived. For stability analysis the Bounded Phase Condition of robust control has been used to determine the parametric stability margin that preserves closed loop stability. The simulation results of both the examples agree with the mathematical conditions derived and illustrate that the design offers robustness to the actual plant in the presence of wide plant parameter variations. This method may be suitable in applications of control systems where tracking, disturbance rejection and robustness are crucial.

The investigation of the method, with more realistic situations including nonlinearities and time delays is left for future work.

References

Åström, K. J. and B. Wittenmark. *Adaptive Control*, Pearson Education, Second Edition, 2006.

Arbogast, E. J. and D. J. Cooper. Extension of IMC tuning correlations for non-self regulating (integrating) processes. *ISA Trans.* , 46:303–311, April, 2007.

Bhattacharyya, S. P., H. Chapellat and L. H. Keel. In Thomas Kailath, editor, *Robust Control:The Parametric Approach*, Prentice Hall, New York, USA, 1995.

Bhusnur, S. and S. Ray Analysis and Comparison of Single Loop and Robust Two Loop Control Structures as Applied to a Position Servo System. *Proc. International Conference on Advances in Control and Optimization of Dynamical Systems (ACODS2007)*,IISc. Bangalore, India, 2007.

Hamamci, S. E. and M. Koksal. Robust Control of A DC motor by Coefficient Diagram Method. *Proc. MED'01 The 9th Mediterranean Conference on Control and Automation*, Dubrovnik, Crotia, June, 2001.

Hamamci, S. E. Kaya, I. and M. Koksal. Improving Performance For A Class of Processes Using Coefficient Diagram Method. *Proc. of 9th IEEE Mediterranean Conference on Control and Automation*, 2001.

Heong, K. Chong, G. and Yun. PID Control System Analysis, Design, and Technology. *IEEE Trans. on Control Systems Technology*, 13:559–576, July, 2005.

Kim, Y.C. and S. Manabe. Introduction to coefficient diagram method. *Proc. of SSSC'01*, Prague, 2001.

Kim, Y. C. Keel, L. H. and S. Manabe. Controller Design for Time Domain Specifications. *Proc. IFAC, 15th Triennial World Congress* , Barcelona, Spain, 2002.

Koksal, M. and S.E. Hamamci. A program for the Design of Linear Time Invariant Control Systems:CDMCAD. *Inc. Comput appl Eng Educ*, Wiley Periodicals,12:165–174, 2004.

Manabe, S. A Solution of The ACC Benchmark Problem By Coefficient Diagram Method. *Proc. 2nd Asian Control Conference*, July, Seoul, 1997.

Manabe, S. Coefficient Diagram Method. *Proc. 14th IFAC Symposium on Automatic Control in Aerospace*, August 1998. Seoul, Korea.

Manabe, S. Sufficient condition for stability and instability by Lipatov and its application to the coefficient diagram method. *Proc. of 9-th Workshop on Astrodynamics and Flight Mechanics*, Sagamihara, ISAS, July, 1999.

Manabe, S. and Y.-Chol Kim. Recent Development in coefficient diagram method. *Proc. 3rd Asian Control Conference*, Shanghail, July, 2000.

Numsomran, A. Witheepanich, K. Tipsuwanporn, V. and N. Klinsmitth Robust Controller Design for Plant Uncertainty. *Proc.-SICE-ICASE International Joint Conference*, October 2006, Busan, Korea.

Osypiuk, R. Kroger, T. Finkemeyer, B. and F. M. Wahl. A Two-Loop Implicit Force/Position Control Structure, Based on a Simple Linear Model: Theory and Experiment. *Proc.- IEEE International Conference on Robotics and Automation* , May, Florida, 2006.

Shunji Manabe. Importance of Coefficient Diagram in Polynomial Method. *Proc. 42nd IEEE Conference on Decision and Control*, Maui, Hawaii, USA, December, 2003.

Shunji Manabe. Case Studies of coefficient diagram method- Practical Polynomial Design Approaches. *Proc. of 16th IFAC World Congress*, Prague, 2005.

Skoczowski, S. Domek and K. Pietrusewicz Model Following PID Control System. *Kybernetes*, 32(5/6):818–828, 2003.

Skoczowski, S. Domek, K. Pietrusewicz and B. Broel-Plater. A Method for Improving the Robustness of PID Control. *IEEE Trans. on Industrial electronics*, 52:1669–1676, 2005.

Stefani, Shahian, Savant and Hostetter. In Thomas Kailath, editor, *Design of feedback control systems*, Oxford University press, fourth edition,2002.

Tsang, K. M. and G. Li. Robust nonlinear nominal model following control to overcome deadzone nonlinearities. *IEEE Trans. on Industrial Electronics*, 48: 177–184, Feb, 2001.

Yhanphairoje, S. Witheepanich, K. Piyarat, W. and S. Intajag. A Robust Control Struc ure Enhancing Nominal Model Following Scheme for Induction Servomotor Drive. *Proc.-SICE Annual Conference in Sapporo*, August, Hokkaido Institute of Tecnology, Japan, 2004.