Robust design with uncertain data and response surface approximation

Wolfgang Graf, Jan-Uwe Sickert, Stephan Pannier and Michael Kaliske
Institute for Structural Analysis, Technische Universität Dresden, Germany,
wolfgang.graf@tu-dresden.de

Abstract. The aim of the presented approach is to combine two views on robustness within a comprehensive design approach. These two main views are on the one hand the resistance to extraordinary (unforeseen) events, and on the other hand the consideration of uncertainty in structural parameters in order to monitor and reduce the variation of structural responses. To capture this, an enhanced robustness measure is defined and applied as design objective within the design approach which is based on the solution of an optimization problem. In addition to sophisticated numerical procedures to map physical phenomena and processes, the adequate description of available data covering the content of provided information is of prime importance. Thus, the generalized uncertainty model fuzzy randomness is applied. Furthermore, the robustness measure as well as the optimization approach are enhanced to fuzzy random based solutions.

Keywords: robustness; imprecise probabilities; fuzzy random optimization; structural analysis; response surface approximation

1. Introduction

In general, design is understood as selection process in order to obtain the geometry of a structure and the material to be applied in terms of design variables. The selection is guided by the general purpose of the structure which defines the requirements. One key requirement is the resistance against all stresses, that includes all kinds of time-dependent loads, environmental conditions, chemical and biological influences, in order to ensure reliable structural behavior. Thereby, structural analysis is performed to prove that the structure will fulfill its intended function for a given load scenario.

Simple structural models with elastic material behavior may be analyzed straightforward by means of closed form functional relations. Against this, the behavior of complex structures is influenced by different nonlinearities, load redistribution and multi-faceted failure modes. This motivates the design approach presented in this contribution which bases on numerical structural analysis and optimization.

Beside sophisticated numerical procedures to map physical phenomena and processes, an adequate description of available data covering the content of provided information is of prime importance. Applying imprecise probabilities, objective components of the uncertainty as well as subjective components can be considered simultaneously, see (Möller and Beer, 2008). A sophisticated procedure to handle imprecise probabilities provides the generalized uncertainty model fuzzy

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randomness. Thereby, well established algorithms for reliability-based design are expandable to incorporate fuzzy random quantities, see Section 3.

In addition to the reliability based design, engineers are confronted with the task to design structures which ensure robust structural behavior during their lifetime. This succeeds by paying attention to qualitative construction rules, such as introducing redundancies and ensure ductility. Nevertheless, quantitative assessments are common part of scientific and practical work in order to reflect the degree of robustness. Even so, no general definition is yet available. Different views on robustness and the meaning of robust design are summarized in Section 2. The aspect of robustness can be incorporated into the design process for instance by means of the fuzzy random optimization approach, see Section 3.3. The applicability of the developed algorithms is demonstrated in Section 5.

2. Concepts of robust design

The aim within a design process is to determine a set of design variables in such a manner, that the analyzed result is superior to all other possible design variants. This is called an optimal design. Further, the design of robust structures focuses mainly at the ability to survive unforeseen events (Knoll and Vogel, 2009), the prevention of progressive failure in the case of local effects (Starossek, 2009), and the safeguard against perturbations due to uncertainty (Fuchs and Neumaier, 2008). Comparing suggested approaches of robust design, one can differentiate between qualitative and quantitative concepts. A further distinction is given due to the application of deterministic and uncertainty based robustness measures.

Qualitative concepts according to e.g. (Pötzl and Schlaich, 1996; Starossek, 2009; Knoll and Vogel, 2009) are based on a collection of aspects, methods and strategies which should be considered in the design process. In (Knoll and Vogel, 2009) qualitative concepts are summarized under the headlines: strength, structural integrity and solidarization, second line of defense, multiple load path or redundancy, ductility, zipper stopper to prevent progressive failure, capacity design, sacrificial and protective devices, knock-out scenarios, stiffness considerations, strain hardening, post buckling resistance, warning and active intervention, testing and monitoring, quality control as well as mechanical devices.

Some of these strategies, such as the consideration of plasticity, strain hardening and post buckling behavior, are utilized today in order to design lightweight structures. They are then inappropriate to safeguard against progressive failure in the case of unforeseen circumstances.

Other strategies are inappropriate to compare diverse designs because they only define construction rules lexically. A measure of robustness is missing so far. Therewith, these concepts are incompatible with approaches in which an optimization problem is solved. In general, the effectiveness of qualitative strategies can only be proven by means of a quantitative assessment of robustness under consideration of uncertainty. Negating the inherent uncertainty in the parameters, the respective design does not ensure a robust structural behavior in any case. In conclusion, incorporating uncertainty within a design process is a must to make use of the results.

Traditionally, a stochastic description is applied in many references to model the uncertainty of excitation and resistance variables, see e.g. (Jensen et al., 2009). Then, the stochastic variables are processed within a reliability analysis, obtaining the failure probability. The failure probability is considered as constraint in order to ensure prescribed safety levels. These approaches may also be referred to as reliability-based design approaches, see e.g. (Jurecka, 2007; Lee et al., 2008). Applying an optimization task as design procedure, a reliability-based design optimization is obtained.

Another view on robust design is based on the recognition of the epistemic uncertainty and the information deficit concerning the stochastic description. That means, in many cases the distribution and their parameters are unknown. Thus, both can only be determined by subjective assessment. This is especially reasonable if the amount of data is limited, the data are obtained under changing reproduction conditions and data are imprecise. In order to ensure robustness anyway, different combinations of parameters are presumed and the results are compared by means of failure probability, (Jensen et al., 2009).

A generalized approach is based on the modeling of epistemic uncertainty by means of non-stochastic uncertainty models. The capabilities of such non-traditional models are summarized in (Möller and Beer, 2008). In result, imprecise probability concepts are formulated, which provide the basis to consider the objective variability of input data as well as the subjective assessment of the available information. The introduction of such concepts into the robust design, e.g., as presented in (Fuchs and Neumaier, 2008), has led to a methodology, which is based on clouds to provide confidence regions for safety constraints in robust design optimization. Further concepts are based on the evidence theory, the p-box approach as well as the theory of interval and fuzzy probability.

The definition of a measure to quantify the robustness within the design task was inter alia pioneered by (Taguchi et al., 2004). The provided measure has been widely applied, e.g. in (Zang et al., 2005; Beyer and Sendhoff, 2007; Beer and Liebscher, 2008), and represents also the basis of the robustness measure defined in Section 3.3. Thereby, the imprecise probability concept fuzzy randomness is applied according to (Möller and Beer, 2004). This generalized uncertainty model contains the special cases based on interval, fuzzy, and random variables which can be considered within the design process simultaneously.

3. Robust-based design optimization

Ordinary design methods are based on an investigation of variants including the proof of a sufficient load-bearing capacity for every variant. In order to improve the efficiency and to ensure a balanced solution between maximizing the utility and minimizing the costs, it has been also formulated as an optimization problem. Furthermore, an inverse solution approach to design of robust structures (Beer and Liebscher, 2008) or solve the lifetime-oriented design problem (Möller et al., 2009) may be applied to determine alternative design margins for preselected designs under consideration of design constraints.

Here, the optimal design is determined by means of an optimization procedure. In an optimization task, the parameters are subdivided into design variables $\underline{x}_d$ and a priori parameters $\underline{p}_a$. Design variables $\underline{x}_d \in \mathbb{X}_d$ are controllable and may be freely selected within user-specified ranges. A priori parameters $\underline{p}_a \in \mathbb{P}$ are predefined and unchangeable in the design process.

3.1. Optimization under consideration of imprecise data

The consideration of imprecise data within the optimization succeeds with the application of the uncertainty model fuzzy randomness. In consequence, design variables $\underline{x}_d \in X$ and a priori parameters $\underline{p}_a \in P$ become fuzzy random variables $\underline{\tilde{X}}_d \in \mathcal{D}\left(\Omega, \mathcal{F}(X)\right)$ and fuzzy random parameters $\underline{\tilde{P}}_a \in \mathcal{D}\left(\Omega, \mathcal{F}(P)\right)$. Introducing fuzzy random input quantities, a fuzzy random objective function

$$
\begin{aligned}
f_{obj}:\ & \mathcal{D}\left(\Omega, \mathcal{F}(X_d)\right) \times \mathcal{D}\left(\Omega, \mathcal{F}(P)\right) \rightarrow \mathcal{D}\left(\Omega, \mathcal{F}(Z)\right); && (1)\\
\text{with} \quad & \mathcal{D}(A, B) : A \rightarrow B; &&\\
& (\underline{\tilde{X}}_d,\ \underline{\tilde{P}}_a) \mapsto \tilde{Z}_r && (2)
\end{aligned}
$$

is obtained. Thereby, Ω is the set of random elementary events and $\mathcal{F}(.)$ is the set of fuzzy quantities.

Within an optimization task uncertain equality and uncertain inequality constraints

$$g_j(\underline{\tilde{X}}_d, \underline{\tilde{P}}_a) \leq \underline{\tilde{G}}_j^*, \quad j = 1,\ \ldots,\ n_g \tag{3}$$

$$h_k(\underline{\tilde{X}}_d, \underline{\tilde{P}}_a) = \underline{\tilde{H}}_k^*, \quad k = 1,\ \ldots,\ n_h \tag{4}$$

may be incorporated. Eqs. (3) and (4) are just symbolic to formulate uncertain constraints. Introducing additional assumptions Eqs. (3) and (4) may be transferred into a mathematical well-defined form, see e.g. Eq. (8).

The challenge of the fuzzy random optimization task is on the one hand to formulate an ordering of fuzzy random quantities in view of alternative design variants within the uncertain objective function and on the other hand to assess the reliability on the basis of fuzzy random quantities. While for ordering of fuzzy quantities a lot of work has been done (Liou and Wang, 1992; Cheng, 1998), approaches for the ordering of fuzzy random quantities are not available to the authors knowledge. Thus, information reducing measures $\mathcal{M}(.)$, which map fuzzy random results to real numbers, are introduced and, thus, the essential ordering in fuzzy random objective and fuzzy random constraints succeeds.

$$\mathcal{M} : \mathcal{D}(\Omega, \mathcal{F}(\mathbb{R})) \rightarrow \mathbb{R} \tag{5}$$

With the aid of $\mathcal{M}(.)$ the minimum of a fuzzy random objective function can be defined with

$$z_{min} = \min_{\underline{\tilde{X}}_d \in \mathcal{D}(\Omega, \mathcal{F}(X))} \left(\mathcal{M}\left(f_{obj}(\underline{\tilde{X}}_d, \underline{\tilde{P}}_a) \right) \right) \tag{6}$$

$$\text{with} \quad \mathcal{M}\left(f_{obj}(\underline{\tilde{X}}_{d,min}, \underline{\tilde{P}}_a) \right) \leqq \mathcal{M}\left(f_{obj}(\underline{\tilde{X}}_d, \underline{\tilde{P}}_a) \right) \ \forall\, \underline{\tilde{X}}_d \in \mathcal{D}(\Omega, \mathcal{F}(X)) \tag{7}$$

under consideration of fuzzy random inequality constraints

$$\mathcal{M}_j^g(g_j(\underline{\tilde{X}}_d, \underline{\tilde{P}}_a)) \leq \mathcal{M}_j^{g^*}(\underline{\tilde{G}}_j^*), \quad j = 1,\ \ldots,\ n_g\ . \tag{8}$$

Naturally, a huge amount of measures $\mathcal{M}$ are available, which assess diverse notions of fuzzy random quantities. Those measures are problem-dependent and have to be specified by the user. Reasonable in the engineering sense is the use of multiple, diverse measures. In consequence, a multi-criteria optimization problem has to be solved.

As elucidated in detail in (Lee et al., 2008), reasonable is to optimize the result of interest and minimize the respective uncertainty. According to this, an appropriate measure is introduced in Section 3.4, which incorporates fuzzy random quantities. Beside solving the announced bi-objective optimization task, further aspects could be launched as objective. Especially in robust design, the need to maximize the reliability may arise instead of fulfilling just a reliability constraint. Furthermore, the design of robust structures forces the engineer to consider on the one hand multiple objectives and on the other hand to cope with diverse load scenarios. On account of solving such a highly multi-objective optimization task, in Section 3.3, a robustness measure, which is formulated to cope with all of the announced aspects, is introduced.

It should be pointed out, that the introduced robust-based optimization approach (Eqs. (6), (8)) is capable to incorporate uncertain and time-dependent variables and parameters. The time-dependency is neglected in the notation on account of the fact, that a time-dependent problem may be transfered into a steady-state problem.

3.2. Optimization and fuzzy stochastic analysis

In the optimization task defined in Eq. (6), an optimization task and the uncertainty characteristic fuzzy randomness are lumped together. A numerical realization succeeds by introducing transformations and assumptions, which enable the decoupling. From a numerical point of view, this enables to constitute a multi-loop algorithm, in which well-established algorithms for optimization, fuzzy analysis and stochastic analysis may be applied.

Especially fuzzy random design variables $\underline{\tilde{X}}_d$ contribute to the optimization and fuzzy stochastic analysis simultaneously. The aim is to formulate $\underline{\tilde{X}}_d$ in such a way, that deterministic design variables are obtained. An applicable procedure utilizes an affine transformation. Thus, $\tilde{X}_d$ can be expressed with $x_{d,1} \cdot \tilde{P}_d + x_{d,2}$. Thereby, $x_{d,1}, x_{d,2} \in X \subseteq \mathbb{R}$ are design quantities within the optimization task and $\tilde{P}_d \in \mathcal{D}(\Omega, \mathcal{F}(\mathbb{R}))$ is an invariant fuzzy random quantity. Due to the fact, that $\tilde{P}_d$ is invariant, it can be considered as a priori parameter. Thus, the a priori parameter vector $\underline{\tilde{P}}_a^* = \left(\underline{\tilde{P}}_a, \underline{\tilde{P}}_d\right)$ is constituted. The optimization task is transformed to

$$
\begin{aligned}
z_{min} = \min_{x_d \in X} \; \left(\mathcal{M}\left(f_{obj}(x_d, \underline{\tilde{P}}_a^*)\right)\right) \qquad (9) \\
\text{with} \quad \mathcal{M}_j^g(g_j(x_d, \underline{\tilde{P}}_a^*)) \leq \mathcal{M}_j^{g^*}(\underline{\tilde{G}}_j^*), \quad j = 1, \ldots, n_g \,.
\end{aligned}
$$

The numerical treatment of fuzzy random quantities succeeds utilizing a bunch parameter representation as elucidated in (Möller and Beer, 2004; Graf et al., 2007).

$$
\underline{\tilde{P}}_a^* = \underline{P}_a^*(\underline{\tilde{s}}) = \left\{\left(\underline{P}_{a,j}^*, \mu\left(\underline{P}_{a,j}^*\right)\right) \mid \underline{P}_{a,j}^* = \underline{P}_a^*(\underline{s}_j);\; \mu\left(\underline{P}_{a,j}^*\right) = \mu\left(\underline{s}_j\right) \forall\, \underline{s}_j \in \underline{\tilde{s}}\right\} . \qquad (10)
$$

Thereby, $\underline{P}_{a,j}^* \in \mathcal{D}(\Omega, P)$ is denoted as original of $\underline{\tilde{P}}_a^*$ and $\underline{\tilde{s}}$ are fuzzy bunch parameters. This bunch parameter representation can be applied for the affine transformation of fuzzy random design variables $\tilde{X}_d$, also. For instance, the fuzziness of $\tilde{X}_d$ may be represented by the two fuzzy bunch parameters: fuzzy mean value $\tilde{\mu}_X$ and fuzzy standard deviation $\tilde{\sigma}_X$. Then, $\tilde{X}_d$ is transformed to $\tilde{X}_d \approx f_d(\tilde{\mu}_X, \tilde{\sigma}_X)$ with $\tilde{\mu}_X = x_{d,1} \cdot \tilde{\mu}_E + x_{d,2}$ and $\tilde{\sigma}_X = x_{d,3} \cdot \tilde{\sigma}_E + x_{d,4}$. However, the application of four design variables $x_{d,1}, \ldots, x_{d,4}$ to describe one physical quantity is inappropriate for engineering application. Reasonably, one discrete design variable x_d should be chosen, wisely $x_d = x_{d_2}$ and the other design variables should be expressed as $x_{d_1} = f_1(x_d)$; $x_{d_3} = f_3(x_d)$; $x_{d_4} = f_4(x_d)$. The functions f_1, f_3 and f_4 are user-specified and can be chosen arbitrarily.

In result, Eq. (6) is decoupled such that we are able to assign to each crisp design point $\underline{x}_d \in X$ a result $\mathcal{M}(\underline{\tilde{Z}})$. Thereby, it is further possible to assess the randomness $\underline{Z}$, e.g., determining failure probabilities or lifetime, and fuzziness $\tilde{z}$ in $\underline{\tilde{Z}}$ separately.

3.3. Robustness as design objective

The robustness of a structure may be selected as a comprehensive design objective or as one part of a multi-criteria problem. In order to consider the robustness as design objective within an optimization task, an appropriate measure has to be formulated.

The aim of the developed robustness measure, introduced in the following, was to provide a measure which both captures different meanings of robustness simultaneously and is independent of the applied uncertainty model. Here, the requirements are met by the introduced robustness measure

$$R_{l,m}^{[p]} = \mathcal{M}\left(\underline{f}_{obj}\left(\underline{\tilde{X}}_d, \underline{\tilde{P}}_a\right)\right) = \frac{\sum\limits_{k=1}^{n} k_k \cdot P_k\left(M\left(\tilde{X}_{d,k}^{[p]}\right)\right)}{\sum\limits_{i=1}^{l} k_i \left(\sum\limits_{j=1}^{m} k_j \cdot P_{i,j}\left(M\left(f_{obj,i,j}\left(\underline{\tilde{X}}_d^{[p]}, \underline{\tilde{P}}_a\right)\right)\right)\right)} \tag{11}$$

with

$R_{l,m}^{[p]}$	...	robustness measure of the $[p]$-th structural design under consideration of l load resp. failure scenarios and m objectives
$M(.)$	...	uncertainty measure under consideration of design constraints $\tilde{G}^*$
$\tilde{X}_{d,k}^{[p]}$	...	k-th element of the vector $\underline{\tilde{X}}_d$ of all fuzzy random design variables
$f_{obj,i,j}(.)$	...	j-th single design objective function, e.g., structural response, failure probability, which is computed for the i-th load scenario
$P(.)$	...	penalty function
k	...	weighting factor.

The robustness measure $R_{l,m}^{[p]}$ refers to the $[p]$-th structural design that is a realization of the point $\underline{x}_d^{[p]}$ in the space of design variables. For each design, it evaluates the influence of n fuzzy random design variables on $m \cdot l$ objectives. To reduce the numerical effort, the a priori parameters are neglected because they contribute to $R_{l,m}^{[p]}$ for all structural designs in the same way.

In general, an overall robustness measure is given by the theoretical case $R_{\infty,\infty}^{[p]}$. The more load cases resp. failure scenarios and objectives in form of structural responses are regarded, the merrier is the result. By reason of applicability, however, the number of structural input parameters and objectives is limited to a countable degree. The indexing $R_{l,m}^{[p]}$ points on the quality of the determined robustness. The more load scenarios l are studied and the more result parameters m are considered, the more significant is the received robustness measure.

As Eq. (11) is an overall robustness measure, it contains other robustness measures, e.g., according to (Taguchi et al., 2004),

$$\overset{*}{R}{}_{1,m}^{[p]}= \frac{\sum\limits_{k=1}^{n+1} k_k \cdot P_k\Big(M\left(\tilde{X}_{d,k}^{[p]}\right)\Big)}{\sum\limits_{j=1}^{m} k_j \cdot P_j\Big(M\left(f_{obj,1,j}\left(\underline{\tilde{X}}_d^{[p]}, \underline{\tilde{P}}_a\right)\right)\Big)} \tag{12}$$

as respective special case. Thereby, the number of regarded load scenarios is limited to $l = 1$ and the uncertainty is quantified by means of variances. In (Taguchi et al., 2004), robustness is mainly defined by reduction of variation. It is postulated, that structural responses resp. production results may be easily adapted to design objectives in the next manufacturing step. For other engineering tasks, a design process, in which design objectives and constraints can be considered, is performed only once. Hence, uncertainty measures, defined in Section 3.4, for the here introduced robustness assessment, capture design targets z_d, e.g., economic aspects, and design limitations z_{lim}, e.g., constraints, together with the variations.

3.4. Uncertainty measures

The uncertainty measures M for fuzzy random variables, as applied in Eq. (11), is defined reasonably on the basis of fuzzy bunch parameters, see Section 3.2. Thereby, a fuzzy random variable $\tilde{X}$ can be described as a random variable with w fuzzy bunch parameters $(\tilde{s}_1, \ldots, \tilde{s}_w)$, e.g. fuzzy mean value $\tilde{\mu}_X$, fuzzy standard deviation $\tilde{\sigma}_X$ or further moments. On account of a bunch parameter representation, the uncertainty of a fuzzy random variable could be determined by

$$M\left(\tilde{X}\right) = \sum_{j=1}^{w} M_i^{DC}\left(\tilde{s}_j\right) \ . \tag{13}$$

Therewith, the uncertainty of $\tilde{X}$ is measured by the uncertainty of the fuzzy bunch parameters

$$M_i^{DC}\left(\tilde{s}_j\right) = \frac{M_i\left(\tilde{s}_j\right)}{M_i\left(\tilde{s}_{j0}\right)} + \frac{z_{DC}\left(\tilde{s}_j\right)}{z_{DC}\left(\tilde{s}_{j0}\right)} \tag{14}$$

defined under consideration of design objectives resp. constraints. Eq. (14) agrees with the definition of the bi-objective optimization problem for random design variables cited by (Lee et al., 2008). For fuzzy variables, the distance measure $z_{DC}\left(\tilde{s}_j\right)$ represents the performance in the mean whereas the uncertain measure $M_i\left(\tilde{s}_j\right)$ appraises the variation of $\tilde{s}_j$.

A variety of measures $M_i\left(\tilde{s}_j\right)$ is available to assess the uncertainty of a fuzzy quantity $\tilde{s}_j$, see e.g. (Wu and Mendel, 2007). Here, the three measures M_1, M_2 and M_3 are picked out.

Among other references, in (Möller and Beer, 2004; Beer and Liebscher, 2008) a measure is introduced which adapts SHANNON's entropy to evaluate the uncertainty of fuzzy quantities. On this basis, the uncertainty measure $M_1 = H_u$ of a continuous fuzzy quantity $\tilde{s}$ is defined by

$$M_1\left(\tilde{s}\right) = H_u\left(\tilde{s}\right) = -\frac{1}{\ln 2}\int_{\min s}^{\max s}\left(\mu\left(s\right)\cdot\ln\mu\left(s\right) + \left(1-\mu\left(s\right)\right)\cdot\ln\left(1-\mu\left(s\right)\right)\right)ds\ . \tag{15}$$

In (Wu and Mendel, 2007), the variance (second central moment) V of a fuzzy variable $\tilde{s}$ is introduced

$$M_2\left(\tilde{s}\right) = V\left(\tilde{s}\right) = \int_{\min s}^{\max s}\left(s-\bar{s}\right)^2\cdot\mu\left(s\right)ds\cdot\left(\int_{\min s}^{\max s}\mu\left(s\right)ds\right)^{-1}. \tag{16}$$

as an alternative uncertainty measure. In contrast to entropy H_u, the variance V of a fuzzy quantity is not evaluating the unsureness of possible elements in $\tilde{s}$, but rather the spread resp. scattering of the fuzzy number itself. In Eq. (16), the centroid (first moment)

$$\bar{s} = \int_{\min s}^{\max s} s\cdot\mu\left(s\right)ds\cdot\left(\int_{\min s}^{\max s}\mu\left(s\right)ds\right)^{-1} \tag{17}$$

of a fuzzy variable $\tilde{s}$ is used.

Another uncertainty measure applies the area (zeroth moment) A of a fuzzy variable $\tilde{s}$. This measure considers the spread of the whole fuzzy number as well. The uncertainty measure $M_3 = A$ of a continuous fuzzy quantity $\tilde{x}$ is defined by

$$M_3\left(\tilde{s}\right) = A\left(\tilde{s}\right) = \int_{\min s}^{\max s}\mu\left(s\right)ds. \tag{18}$$

With aid of the three measures entropy $M_1\left(\tilde{s}\right)$, variance $M_2\left(\tilde{s}\right)$ and area $M_3\left(\tilde{s}\right)$, the uncertainty of fuzzy bunch parameters $M_i\left(\tilde{s}_j\right)$ in Eq. (14) can be evaluated. In order to reduce the dimensionality problem of the two objectives in Eq. (14), the uncertainty measure is normalized by the means of $M_i\left(\tilde{s}_{j0}\right)$.

The distance measure $z_{DC}\left(\tilde{s}\right)$ representing the performance in the mean depends on the kind of restriction regarded. Two cases are suggested.

1. Consideration of a design target:
   Thereby, the distance from the design target s_d to the centroid $\bar{s}$ of the fuzzy number is analyzed.

$$z_{DC}\left(\tilde{s}\right) = z_d\left(\tilde{s}\right) = \left|s_d - \bar{s}\right| \tag{19}$$

2. Consideration of a design constraint:
   Thereby, the distance from the design limitation s_{lim} to the upper end of the support $s_{\alpha=0,r}$ of the fuzzy number $\tilde{s}$ is analyzed.

$$z_{DC}\left(\tilde{s}\right) = z_{lim}\left(\tilde{s}\right) = \frac{1}{s_{lim} - s_{\alpha=0,r}} \tag{20}$$

In Figure 1, structural responses $\tilde{z}_i$ of three different designs $i = 1, 2, 3$, one design objective z_d and one design constraint z_{lim} are shown. Considering only the uncertainty of the structural responses, $\tilde{z}_3$ is the most uncertain and, in this sense, the least robust response. However, if z_{lim} is for instance a failure criterion, $\tilde{z}_3$ has the largest reserves in bearing capacity. Measuring the uncertainty of $\tilde{z}_1$ and $\tilde{z}_2$ results in equal values. Nevertheless, as $\tilde{z}_2$ is farther away from the failure criterion, it is assessed as more robust.

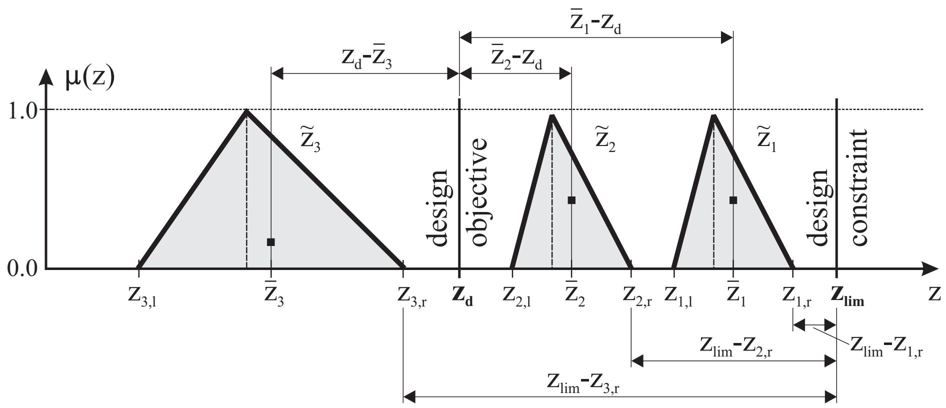

Figure 1. Consideration of design objectives and constraints

The generalized uncertainty model fuzzy randomness contains fuzziness and randomness as special cases. The uncertainty measure for fuzzy random variables defined in Eq. (13) covers these special cases as well.

4. Response surface approximation with neural networks

By reason of applicability for complex problems, the introduced optimization task for fuzzy random quantities in Section 3.1 should be formulated in a numerically manageable manner and reasonable in engineering sense.

An optimization task under consideration of uncertain input quantities is always cumbersome especially in application for complex problems. On account of a hierarchical numerical algorithm, the deterministic solution $f(x)$ has to be repeated very often. This forces the computational time up to an unreasonable extent.

An appropriate measure to improve the numerical efficiency is the approximation of the response surface, constituted by the deterministic solution $f(x) = z$, with the aid of a surrogate model $f^*(x) = z^*$. On the basis of such a surrogate model, the response surface is evaluated approximately in a numerical cheap manner and, thus, ensures the applicability. A survey about response surface approximation schemes, so called metamodels, can be found, e.g., in (Simpson et al., 2001).

The general scope to build a metamodel consists of a three step procedure. Firstly, a set of points $\{(x_1, z_1), \ldots, (x_i, z_i), \ldots, (x_n, z_n) \mid i = 1, \ldots, n\}$ is generated with the aid of the deterministic solution f. Next, a capable surrogate model f^* has to be chosen, i.e a neural network. Thirdly, the surrogate model f^* is fitted to the initially determined point set, such that the characteristic of f is approximated appropriately. Before the constituted metamodel is applied within the optimization task, the appropriateness of the approximation quality should be validated.

A superior metamodel utilizes artificial neural networks, see (Haykin, 1999). The idea of artificial neural networks is based on the design of the human brain. The human brain is constituted by information-processing units (so called neurons) that are connected by synapses, and it forms the kernel of the human nervous system. It is capable of processing input signals that are derived from the environment and of providing appropriate output signals (e.g. certain actions). The advantages of the human information processing system are complexity, nonlinearity, and parallelism. An artificial neural network resembles the human brain in many respects. It is constituted by neurons which are connected by synapses, it has the ability of mapping input signals onto output signals and to adapt to initial data during a training phase. On account of those properties, a neural network can replace the deterministic solution $f(x)$ as a surrogate model $f^*(x)$. That is, the input signals comprise structural input quantities x and the network output provides the associated result of interest z.

The neural network firstly needs to learn the features of the underlying response surface f on the basis of initially determined deterministic solution $\{(x_1, z_1), \ldots, (x_i, z_i), \ldots, (x_n, z_n) \mid i = 1, \ldots, n\}$. The selection of appropriate initially determined points is called design of experiments (DOE). Therefore, arbitrary schemes for numerical experiments are available, see (Sacks et al., 1989). The aim is an efficient placement of the predetermined points to obtain as much as possible information on the basis of a minimal number of points. Of course, for complex problems with a non-monotonic structural behavior, each structural analysis $f(x)$ is a gain in information.

A proper choice of a neural network as surrogate model is problem-dependent and an optimization task itself. Feedforward neural networks are suitable for optimization tasks with uncertain quantities. A multi-layer feedforward neural network (see Fig. 2) permits signal flow exclusively in forward direction through the network. The neurons are organized in different layers: one input layer, one or more hidden layers, and one output layer. The produced output y of each neuron of a layer is transmitted to one or more neurons of the following layer by synaptic connections. Within a fully connected network, every neuron is linked to all neurons of the following layer. Otherwise, if several connections are missing, the neural network is referred to as partially connected. Furthermore, it is possible to introduce shortcut connections between neurons of non-adjacent layers. The constitution of the multi-layer feedforward neural network may be determined by means of a network-topology optimization. Thereby, an appropriate constitution of the neural network, number of hidden layers and number of neurons per hidden layer, is determined.

The set of signal input/output to train neural networks the characteristic of f is provided by standardizing the input quantities $x_i, i = 1, \ldots, n$ and normalizing the result quantities $z_i, i = 1, \ldots, n$. On the basis of those signals, the neural network is trained by adjusting the synaptic weights. The synaptic weights may be interpreted as adjustable free parameters of a neural network and have the task to strengthen or weaken the signals transferred by the synaptic connections. The knowledge represented by a neural network after the training is stored in its synaptic weights. For the training of neural networks, various methods are on hand. One training algorithm is the backpropagation algorithm, see (Haykin, 1999), utilizing the differentiability of the neural network. Furthermore, different genetic algorithms are available.

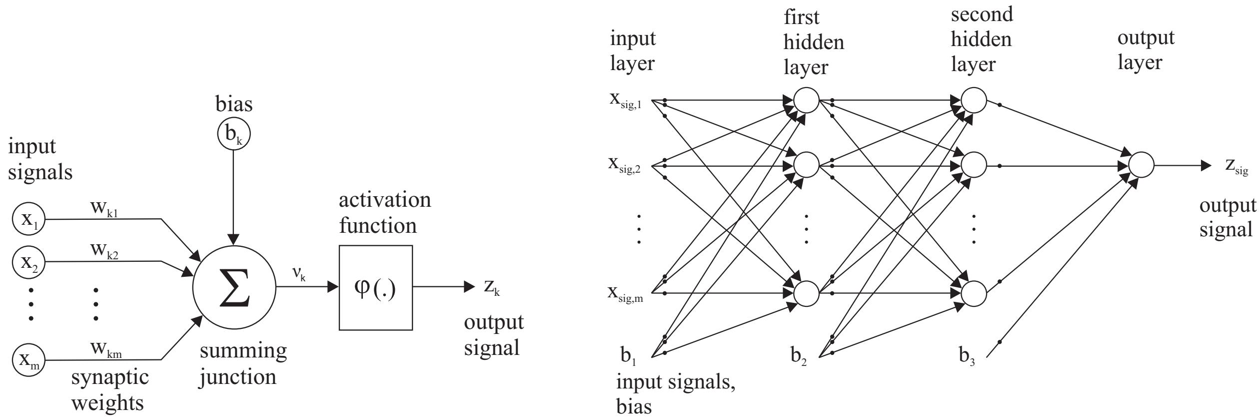

Figure 2. Constitution of a neural network; construct of a neuron (left) and feedforward neu al network (right)

The trained neural network has to be validated, before it is applied in the optimization task, with a virgin point set $\{(x_1, z_1), \ldots, (x_j, z_j), \ldots, (x_m, z_m) \mid j = 1, \ldots, m\}$. Thereby, the appropriateness is evaluated by means of a relative error. If the quality is insufficient, the training may be repeated with modified neural network topologies. If the quality remains insufficient, further points of $f(x)$ have to be determined to enlarge the training data.

For complex problems, the generation of initially determined point sets of $f(x)$ is limited to a minimal extend to comply with economical aspects. Thus, measures to improve the approximation quality of metamodels are required. This succeeds by combining single neural networks to network machines.

A well known measure is the constitution of so called committee machines. They are formulated with a parallel structure of individual neural networks f_k^* on the basis of the same input-output pairs $\{(x_1, z_1), \ldots, (x_i, z_i), \ldots, (x_n, z_n) \mid i = 1, \ldots, n\}$. Due to the fact, that specifying diverse network topologies and repeating the neural network training several times show random effects, a response surface approximation with neural networks is generally not unique. This effect is amplified if the underlying data exhibit noise. It is reasonable to finally eliminate those random effects by averaging the outputs $z_k^* = f_k^*\left(\underline{x}\right)$ of the individual networks f_k^* of the committee machine

$$z^* = \frac{1}{n} \sum_{i=1}^{n} z_k^* \; . \tag{21}$$

Another approximation scheme, introduced by (Liebscher, 2007), focuses on the simultaneous representation of local and global function features. A first neural network f_1^* is trained to capture the global trend of the response surface reflected in the initial training data set. Then, the approximation error is computed $\underline{e} = \underline{z}_i - \underline{z}_1^*$. The error surface primarily reflects local function features, which have not been captured by the first network f_1^*. A second network f_2^* is trained on the basis of the error surface with the data set $[\underline{x}_i;\, \underline{z}_i - \underline{z}_1^*]$ and can concentrate on the local function features only. The sum of the network outputs then yields an improved approximation (Eq. (22)). This scheme may be applied further to sequentially reduce the approximation error. Therefore, the training data set is updated for each network f_k^* to $\left[\underline{x}_i;\, \underline{z}_i - \sum_{q=1}^{k-1} \underline{z}_q^*\right]$. The result

$\underline{z}^*$ is obtained as the sum of output $z_k^* = f_k^*(\underline{x})$

$$z^* = \sum_{k=1}^{n} z_k^* \tag{22}$$

of the individual networks f_k^*. A further measure to improve the approximation quality is the patchwork approximation approach (Pannier et al., 2009). Thereby, neural networks are applied section-wise on local parts of the response surface. The design of a patch is not bounded to any requirements. It may be best constituted in dependence of the respective problem and the available set of input-output pairs. The approximation of the response surface within a patch may be performed with arbitrary approximation schemes. Generally, the behavior of local parts of the response surface is less complex than the function features of the complete response surface. Thus, the requirements onto an approximation scheme of local function features is less rigorous. To preserve a high degree of generality and flexibility, an application of neural networks is reasonable. If the supporting points of a patch-network are determined in combination with the patchwork approximations, an beneficial union is established. This is referred to as an integrated patchwork approximation. However, for an predefined set of input-output pairs a pure patchwork approximation is also enabled.

In an integrated patchwork approximation, the design of experiments is adjusted to the requirements of the patch-network. In dependence of the required approximation quality, the number of input-output pairs is predefined. Furthermore, the constitution of the patch-size is adjustable to the respective problem. Generally, the user-defined specifications may increase the approximation quality on the one hand, but on the other hand, it always augments the number of deterministic solutions f. The size of a patch may be determined in dependence of the initial size of the input space $\Delta\underline{X}$, see Eq. (23). Thereby, $\underline{a}$ scales the size of the patch and n denotes the number of individual patch-networks.

$$\underline{p}_i = \underline{a} \cdot \Delta\underline{X} \mid i = 1, \ldots, n \tag{23}$$

In order to optimize the local approximation quality, the patches are specified online so that the point of interest for the approximation in each case coincides with the center of gravity of the patch. If a patch does not contain a minimum number of input-output points, the point of interest is evaluated with the aid of the deterministic fundamental solution f. This leads to a moderate supplementation of the available set of input-output pairs in sparsely populated domains. Due to an integrated determination of the supporting points for a patch-network, the number of input-output pairs is reduced to a minimal amount.

A pure patchwork approximation utilizes an available set of input-output pairs. In the neighborhood of the point of interest, a predefined number of input-output pairs, which are closest to the point of interest, are determined with the aid of the Euclidean distance. Thus, the patch-size depends on the distribution of the input-output pairs. The point of interest is obligatory not the center of gravity. A point of interest, situated in a sparsely populated domain, is analyzed without an adequate set of input-output pairs.

In view of the numerical efficiency of patchwork approximation, the limitation of the network approximation to small patches and subsets of the initial data enables the application of small neural networks. Consequently, the training is associated with a low numerical expense. Otherwise, within a patchwork approximation, the neural networks have to be trained several times. The repetition of the training is linked to an increase of the computational expense. This fact becomes important if a high number of points have to be evaluated, e.g., in reliability analysis utilizing Monte Carlo simulation. The numerical efficiency can be improved by re-using patch networks for further points of interest. Thereby, it may be demanded, that the new point of interest $\underline{x}$ have to be situated, in relation to the center of gravity of a still trained patch-network p_i, within a predefined distance.

5. Examples

5.1. Optimization of a deep drawing process

In this example, an appropriate design of a metal forming process, see Fig. 3, should be determined. The aim is to identify a setting of design quantities whose results comply with reliability requirements in a best possible manner.

Figure 3. Metal forming device and component part

Among all input quantities, sixteen are indicated to be sensitive to the result quantities and, thus, to influence the reliability predominantly, elucidated in detail in (Müllerschön et al., 2007). These are material parameters for a blank to describe the yield strength, the elasto-plastic hardening and anisotropic effects. In detail, these are the parameters of the swift law R_p, n, K and the anisotropic coefficients r_0, r_{45}, r_{90}. Furthermore, variations of the manufacturing process parameters, like the friction coefficient μ, the draw bead forces and the binder forces, affect the performance of the deep drawing process. At least, spatial perturbations of the initial shell thickness, caused by the production process of the blank itself, have to be considered. They are modeled with the aid of fuzzy functions (Möller and Beer, 2004).

Design quantities $\underline{x}_d$ are the mean value of the binder force $x_{1,d}$ and the mean values of the draw bead forces $x_{2,d}$, $\ldots$, $x_{7,d}$, see also Section 3.2. The respective design ranges are predefined with intervals as follows: $x_{1,d}$ = [1400, 2400], $x_{2,d} = x_{3,d} = x_{7,d}$ = [20, 200], $x_{4,d}$ = [50, 120], $x_{5,d}$ = [60, 120], $x_{6,d}$ = [70, 130]. A priori input quantities are the invariant a priori parameters itself and the decoupled uncertainty part of design quantities. The quantification of the uncertain input quantities is accomplished under consideration of the respective source of uncertainty. Hence, the generalized uncertainty model fuzzy randomness is utilized, see Table I. Thereby, it is assumed that the randomness in respective input parameters can be modeled by a normal distribution. The fuzzy parameters are modeled as fuzzy triangular numbers quoted by $< s_l, t, s_r >$, whereby the interval $[s_l,\ s_r]$ describes the support and t defines the value with the highest membership.

Table I. Fuzzy random input quantities ($\tilde{f} : \Omega \rightarrow \mathcal{F}(X)$)

fuzzy random quantity		normal distribution mean value	standard deviation
yield strength f_y	$\tilde{f}(x_1)$	0.14	$< 0.0067; 0.008; 0.01 >$
strength coefficient K	$\tilde{f}(x_2)$	0.55	$< 0.0367; 0.044; 0.055 >$
hardening exponent n	$\tilde{f}(x_3)$	$< 0.23; 0.275; 0.3 >$	0
friction coefficient μ	$\tilde{f}(x_4)$	$< 0.05; 0.075; 0.1 >$	0
perturbation longitudinal p_1	$\tilde{f}(x_5)$	$< -0.005; 0.0; 0.005 >$	0
perturbation lateral p_2	$\tilde{f}(x_6)$	$< -0.005; 0.0; 0.005 >$	0
material parameter r_0	$\tilde{f}(x_7)$	2.25	$< 0.0833; 0.1; 0.125 >$
material parameter r_{45}	$\tilde{f}(x_8)$	1.7	$< 0.1; 0.12; 0.15 >$
material parameter r_{90}	$\tilde{f}(x_9)$	2.85	$< 0.167; 0.14; 0.175 >$
draw bead force 1	$\tilde{f}(x_{10})$	$\mathrm{x}_{2,\mathrm{d}}$	$< 4.0; 5.0; 6.0 >$
draw bead force 2	$\tilde{f}(x_{11})$	$\mathrm{x}_{3,\mathrm{d}}$	$< 4.0; 5.0; 6.0 >$
draw bead force 3	$\tilde{f}(x_{12})$	$\mathrm{x}_{4,\mathrm{d}}$	$< 4.0; 5.0; 6.0 >$
draw bead force 4	$\tilde{f}(x_{13})$	$\mathrm{x}_{5,\mathrm{d}}$	$< 4.0; 5.0; 6.0 >$
draw bead force 5	$\tilde{f}(x_{14})$	$\mathrm{x}_{6,\mathrm{d}}$	$< 4.0; 5.0; 6.0 >$
draw bead force 6	$\tilde{f}(x_{15})$	$\mathrm{x}_{7,\mathrm{d}}$	$< 4.0; 5.0; 6.0 >$
binder force	$\tilde{f}(x_{16})$	$\mathrm{x}_{1,\mathrm{d}}$	$< 40; 50; 60 >$

The optimization under consideration of fuzzy random quantities is performed by means of generic optimization algorithms, fuzzy structural analysis, and direct Monte Carlo simulation. The application of those methods requires a high numerical effort. Hence, methods to improve the numerical efficiency are inevitable. Here, a response surface approximation on the basis of artificial neural networks, see Section 4, is applied.

As a result of each fuzzy stochastic analysis for a crisp design $\underline{x}_d$, under consideration of $\underline{\tilde{P}}_a^*$, the maximal shell thickness reduction $\tilde{Z}$ is evaluated. The maximal shell thickness reduction represents in a crude way a result to appraise the permissibility of a design. Specifying results of more than 20 % (normalized) as failure, a fuzzy failure probability $\tilde{P}_f$ can be determined.

In the absence of design constraints, the objective is to minimize the largest possible failure probability $P_{f,\alpha=0,r}$. Thereby, an inappropriate variability is penalized.

In the process of optimization, the fuzzy failure probability $\tilde{P}_f$ decreases, see Fig. 4. The optimal design is determined with $\underline{x}_{d,opt} = [25.23; 200.0; 50.0; 60.0; 77.5; 20.0; 1414.81]$. The respective result of the fuzzy stochastic analysis is depicted in Fig. 4 characterized by the fuzzy cumulative distribution function $\underline{\tilde{F}}(\underline{z})$ and the fuzzy failure probability $\tilde{P}_f$.

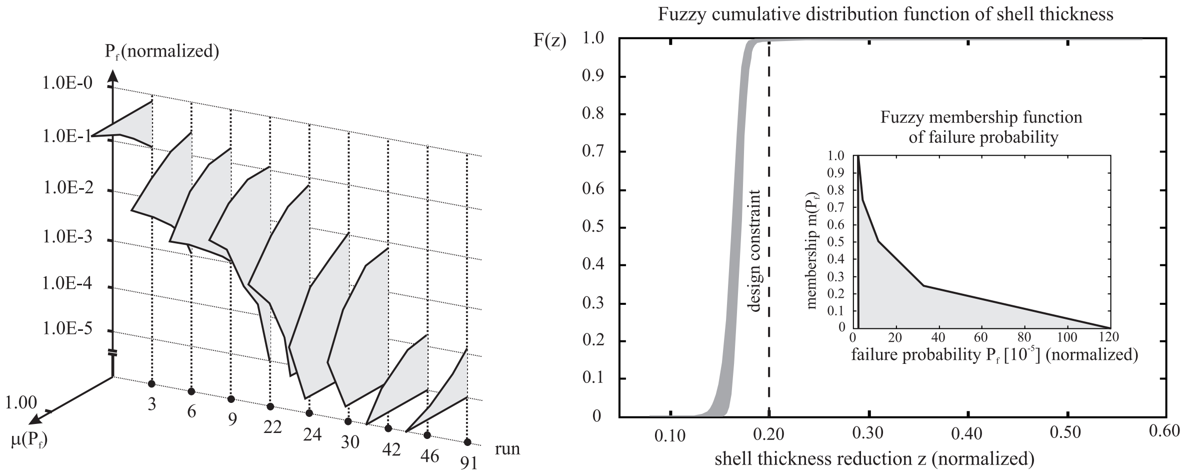

Figure 4. Fuzzy failure probabilities in selected optimization states (left) and (right) result of optimal design (run 91)

In this example, the robustness of the production process for a deep drawing device was optimized. Among all possible design variants, the most robust one was determined on the basis of imprecise data.

5.2. Design of strengthening

The robust design approach is applied to the dimensioning of strengthening for a reinforced concrete (RC) porch roof structure. The aim is to determine a reasonable amount of textile layers, which should be inlaid in a strengthening layer. The porch roof is a part of an existing building and damaged on account of loads and environmental influences. In order to ensure structural safety and reliability as well as robust structural behavior, the porch roof should be strengthened. Representative for arbitrary loads, the determination of the robustness measure is exemplified for wind, snow and two dynamic excitations. Among a plenty of results, the vertical displacements of the porch roof at three characteristic points are computed. The aim of the investigation is to determine the most robust design selected out of five variants.

As strengthening technology, the new composite material textile reinforced concrete (TRC) is chosen. Both on the upside and underside of the porch roof TRC layers may be applied. The TRC layers may consist of up to four multiaxial warp knitted fabrics, so called textiles, placed into a fine grained concrete matrix. These textiles consist of filament yarns (rovings) which are connected with the aid of stitching yarn. Each roving is composed by a lot of single filaments consisting of alcali-resistent glass (AR glass) or carbon.

The load-bearing behavior of RC structures with textile strengthening is appropriately described with the multi-reference-plane model (MRM). The MRM enables to model multi-layered composite materials with a discontinuous multi-Bernoulli-kinematics (Möller et al., 2005; Graf et al., 2007). A MRM element comprises k + 1 layered sub-elements and k interfaces. The sub-element i with its corresponding reference plane RP_i (i = 0, ..., k) is subdivided into s_i sub-layers (concrete and steel sub-layers or fine grained concrete and textile sub-layers). The location of reference plane RP_i

may be selected arbitrarily. Figure 5 displays the FE model selected. Furthermore, in Figure 5 the concrete and reinforcement layers are sketched together with the strengthening layers at the upper and lower surface.

Table II. Uncertain input variables

name	type	parameter 1	parameter 2
f_c (compressive strength) of concrete layers of RP1	lognormal	$\tilde{\mu} =< 28, 30, 32 >$ N/mm^2	$\sigma = 2\ N/mm^2$
f_c of concrete layers of RP 2	lognormal	$\mu = 83\ N/mm^2$	$\sigma = 8\ N/mm^2$
f_c of concrete layers of RP 3	lognormal	$\mu = 83\ N/mm^2$	$\sigma = 8\ N/mm^2$
c of concrete layers of RP 2	normal	$\mu = -1.095$	$\sigma = 0.129$
c of concrete layers of RP 3	normal	$\mu = -0.8$	$\sigma = 0.1$
distributed load p before strengthening	Ex-max-type I	$\mu = 1\ kN/m^2$	$\tilde{\sigma} =< 0.05,\ 0.1,\ 0.2 >$ kN/m^2
a_{s2} (elements 1-20) in Θ_1-direction	shifted exponential	$\tilde{\lambda} =< 7,\ 8,\ 9 >$	$d = 1.13\ cm^2/m$
a_{s1} (elements 1-20) in Θ_1-direction	shifted exponential	$\tilde{\lambda} =< 7,\ 8,\ 9 >$	$d = 1.13\ cm^2/m$
a_{s2} (elements 21-55) in Θ_1-direction	shifted exponential	$\tilde{\lambda} =< 7,\ 8,\ 9 >$	$d = 1.13\ cm^2/m$
a_{s1} (elements 21-55) in Θ_1-direction	shifted exponential	$\tilde{\lambda} =< 1.6,\ 2,\ 2.4 >$	$d = 4.63\ cm^2/m$
a_{s2} (elements 21-55) in Θ_2-direction	shifted exponential	$\tilde{\lambda} =< 7,\ 8,\ 9 >$	$d = 1.09\ cm^2/m$
ρ old concrete	normal	$\mu = 2.3\ t/m^3$	$\sigma = 0.2\ t/m^3$
ρ fine grained concrete	normal	$\mu = 2.3\ t/m^3$	$\sigma = 0.2\ t/m^3$

In order to describe the composite structure comprised of reinforced concrete and textile strengthening, different nonlinear material laws are applied to the individual sub-layers of concrete, steel and textile. Endochronic material laws for concrete and steel are utilized for general loading, unloading, and cyclic loading processes, and taking into account the accumulated material damage during the load history. The endochronic material law for concrete is also adapted to the fine grained concrete. A nonlinear elastic brittle material law is used for the textile reinforcement. Under cyclic loading, damage occurs in the strengthening layer in the fine grained concrete matrix and the textile structure as well as disruption of the bond between the old concrete and the textile strengthening. These kinds of damage phenomena and the additional plastic deformations may be described theoretically by means of plasticity and continuum damage theory. The subsequent strengthening of a RC structure is a structural modification, i.e. a modification of a preloaded and possibly damaged construction. The process of structural modification is simulated numerically by an incremental iterative execution.

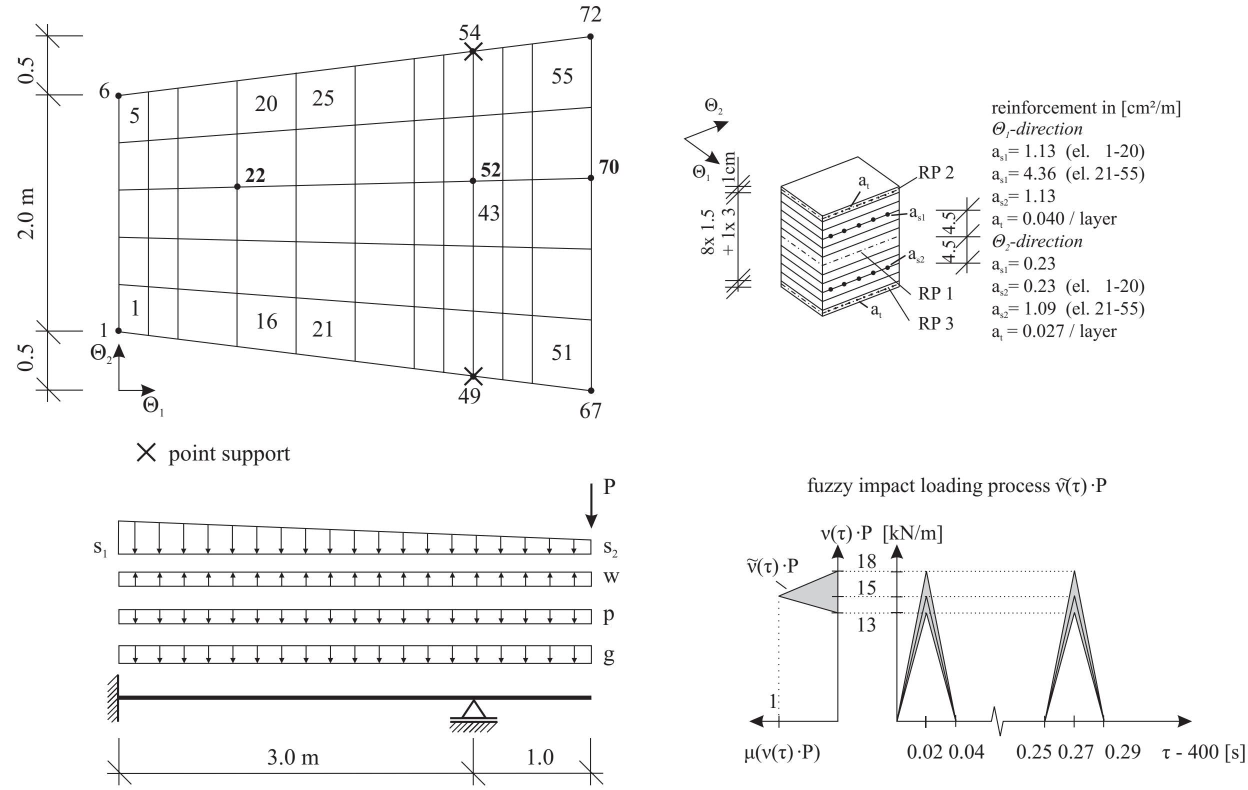

Figure 5. Geometry, FE model, and loading

The introduction of TRC is accompanied by data uncertainty. Aware of the governing uncertainty in the development of TRC structures, e.g. regarding the bond between filaments in textile yarns or the determination of sensitive material parameters, enhanced uncertainty dependent numerical concepts are required to simulate the load-bearing behavior realistically. In the case of plane textile reinforced concrete structures, spatially distributed uncertainty depending on the position vector also arises. This uncertainty may be accounted for using uncertain functions. Furthermore, the uncertain time-dependent material behavior of the textiles and the fine grained concrete requires the formulation of uncertain processes.

TRC structures show data uncertainty of different characteristic and an information deficit exists due to the fact that the boundary conditions are (apparently) subject to arbitrary fluctuations, a comprehensive system overview is lacking, the number of observations are only available to a limited extend, or the sample elements are of doubtful accuracy (non-precise). The outcome of this is a gap between the mathematical quality requirements of data if using stochastic methods and the real available non-precise data. The data do not fully satisfy real valued probability laws. In fact, the data may be quantified by fuzzy probability, see e.g. (Möller and Beer, 2004).

In this example, 13 uncertain input variables are considered. The uncertainty of these parameters is modeled with the aid of fuzzy probability distribution functions; the respective types and parameters are summarized in Table II.

The deterioration of the reinforcement is modeled by means of a reduced cross section. Thereby, the reduction is described by a shifted exponential distribution according to

$$\tilde{F}(x_t) = e^{\tilde{\lambda}(x_t - d)} \tag{24}$$

in dependency of the fuzzy parameter $\tilde{\lambda}$. The random variable c is utilized to determine the concrete tensile strength according to $f_{ct} = (0.3+c) f_c^{(2/3)}$. On account of an uncertain density ρ, an uncertain dead load g is obtained.

The displacement of the porch roof is computed by means of the fuzzy stochastic analysis (FSA), see e.g. (Graf et al., 2007). From a variety of possible strengthening, five variants are compared. The variants are quoted with up/un, whereby up and un indicate the numbers of textile layers embedded in fine grained concrete and applied on the upside and underside respectively, see Table III. In the case of $up = 0$ or $un = 0$ the respective strengthening layer is omitted. The strengthening is applied after a preloading and damaging process. Afterwards, the strengthened structures are exposed to short-term shock loads, snow or wind. The first load process (LP 1) contains two dynamic shock loads acting at the roof tip in vertical direction. The horizontal shock load modeled as concentrated load at node 49 (LP 2) may result from a vehicle impact at the column. Within the snow load process LP 3, a snow slid from the roof of the main building and a snow drift on account of a height difference between the building and the porch roof is considered. The wind load process LP 4 act at the porch roof as upwind. As result of the FSA, the fuzzy bunch parameters of three fuzzy random displacements (FE nodes 22, 52, 70) for the investigated five design variants are computed. On the basis of these results, the robustness measures $R_{1,3}^{[p]}$ are determined applying Eq. (11) for every load case. Therefore, the uncertainty measure $M(.)$, according to Eq. (13), for fuzzy random variables is utilized. Subsequently, the penalty function P(.) is defined by means of a simple quadratic dependency. Figure 6 displays the normalized results.

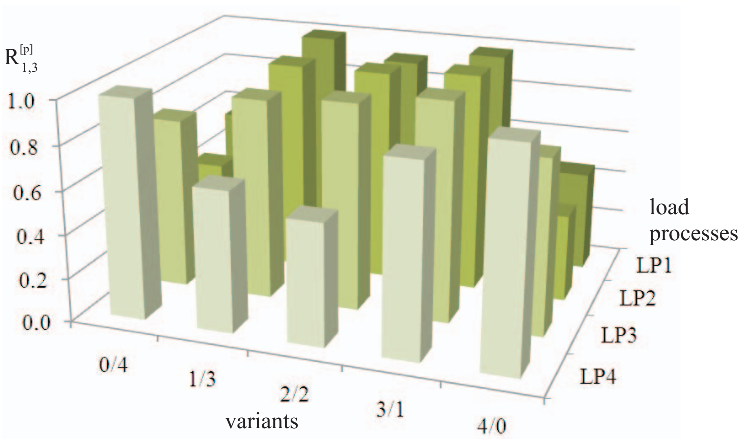

Figure 6. Robustness measures $R_{1,3}^{[p]}$ for every load case using $M_2(\tilde{z})$ according to Eq. (16)

The weighting factors $k_i(.)$, $i = 1, \ldots, 4$ are selected to 0.8 and 0.7 for the dynamic load processes LP1 and LP2, 1.0 for snow (LP3) and 1.1 for wind (LP4). This diverse weighting is based on a subjective assumption.

Table III. Comparison of robustness

p	0/4	1/3	2/2	3/1	4/0
$R_{4,3}^{[p]}$	0.757	0.809	0.721	1.000	0.700

In Table III, the robustness measures $R_{4,3}^{[p]}$ for the five design variants are given. For the uncertainty measures $M(.)$ contained in Eq. (13) the second central moments V according to Eq. (16) are determined to assess the uncertainty within all quantities. Due to the fact, that no design constraints are defined, the uncertainty measures evaluate small displacements as preferable. Under consideration of penalty functions and weighting factors the obtained robustness measures $R_{4,3}^{[p]}$ indicate clearly the design variant 3/1 as most robust.

6. Conclusions

In this paper, a robust design approach is presented. The design is determined by means of an optimization task. The objective is to assess the robustness of different design variants. Thereby, the robustness is evaluated by an overall measure which considers the respective uncertainty. This presumes an adequate modeling of the available information. At this, both aleatoric and epistemic influences are captured. This succeeds by the utilization of a generalized uncertainty model. Here, fuzzy random variables resp. processes are applied. Due to the fact, that the robustness should be described deterministically, measurements for quantifying the uncertainty of fuzzy random variables have been introduced. Beside the variation of input parameters, design constraints are considered. In this approach, uncertainty measures for fuzzy and fuzzy random processes are formulated. The numerical realization of the robust design approach succeeds with the utilization of response surface approximation schemes.

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References

Beer, M. and M. Liebscher. Designing robust structures a nonlinear simulation based approach. *Computers & Structures*, 86(10): 1102-1122, 2008.

Beyer H.-G. and B. Sendhoff. Robust optimization – A comprehensive survey. *Computer methods in applied mechanics and engineering*, 196(33-34):3190–3218, 2007.

Cheng C.-H. A new approach for ranking fuzzy numbers by distance method. *Fuzzy Sets and Systems*, 95(3):307–317, 1998.

England, J., Agarwal, J., and Blockley, D. The vulnerability of structures to unforeseen events. *Computers & Structures*, 86(10):1042-1051, 2008.

Faber, M. and H. Narasimhan. Advances in the evaluation of robustness of structures. In F. Mazzolani, editor, *Urban habitat constructions under catastrophic events - COST Action C26*, Malta, 2008.

Fuchs, M. and A. Neumaier. Uncertainty modeling with clouds in autonomous robust design optimization. In *Proceedings of the 3rd International Workshop on Reliable Engineering Computing*, 1–22, 2008.

Graf, W., A. Hoffmann, B. Möller, J.-U. Sickert, and F. Steinigen. Analysis of textile reinforced concrete structures under consideration of non-traditional uncertainty models. *Engineering Structures*, 29(12): 3420–3431, 2007.

Haykin, S. *Neural Networks: A Comprehensive Foundation.* Prentice Hall, New York, 1999.

Jensen, H.A., G.I. Schuëller, M.A. Valdebenito, and D.S. Kusanovic. An efficient first-order scheme for searching optimal robust designs. In M. Papadrakakis, N.D. Lagaros, and M. Fragiadakis, editors, *2nd Int. Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, Rhodos*, CD-ROM, 2009.

Jurecka, F. *Robust Design Optimization Based on Metamodeling Techniques.* Dissertation, Schriftenreihe des Lehrstuhls für Statik TU München Band 8, Shaker, Aachen, 2007.

Knoll, F. and T. Vogel. *Design for Robustness.* Int. Association for Bridge and Structural Engineering, 2009.

Lee, I., K.K. Choi, L. Du, and D. Gorsich. Dimension reduction method for reliability-based robust design optimization *Computers & Structures*, 86(13-14):1550-1562, 2008.

Liebscher, M. *Dimensionierung und Bewertung von Tragwerken bei Unschrfe — Lösung des inversen Problems mit Methode der explorativen Datenanalyse.* Dissertation, TU Dresden, 2007.

Liou, T.-S. and M.-J. J. Wang. Ranking fuzzy numbers with integral value *Fuzzy Sets and Systems*, 50(3):247-255, 1992.

Möller, B. and M. Beer. *Fuzzy Randomness — Uncertainty in Civil Engineering and Computational Mechanics.* Springer, Berlin, 2004.

Möller, B. and M. Beer. Engineering computation under uncertainty – Capabilities of non-traditional models. *Computers & Structures*, 86(10):1024-1041, 2008.

Möller, B., W. Graf, A. Hoffmann, and F. Steinigen. Numerical simulation of RC structures with textile reinforcement. *Computers & Structures*, 83(19-20):1659–1688, 2005.

Möller, B., M. Liebscher, S. Pannier, W. Graf, and J.-U. Sickert. An inverse solution of the lifetime-oriented design problem. *Structure and Infrastructure Engineering*, DOI: 10.1080/15732470802658946.

Müllerschön, H., D. Lorenz, W. Roux, M. Liebscher, S. Pannier and K. Roll. Probabilistic Analysis of Uncertainties in the Manufacturing Process of Metal Forming. In *6 th European LS-DYNA Users Conference, Gothenburg*, CD-ROM, 2007.

Pannier, S., J.-U. Sickert, and W. Graf. Patchwork approximation scheme for reliability assessment and optimization. In H. Furuta, editor, *10th International Conference on Structural Safety and Reliability, Osaka*, CD-ROM, 2009.

Pötzl, M. and J. Schlaich. Robust concrete bridges without bearings and joints. *Structural Engineering International (IABSE)*, 6(4):266–268, 1996.

Sacks, J., W. J. Welch, T. J. Mitchell and H. P. Wynn. Design and analysis of computer experiments. *Statistical Science*, 4(4):409–435, 1989.

Sickert, J.-U., S. Pannier, W. Graf, and C. Jenkel. Robustness assessment of structures incorporating generalized uncertainty models. In H. Furuta, editor, *10th International Conference on Structural Safety and Reliability, Osaka*, CD-ROM, 2009.

Simpson, T. W., J. D. Peplinski, P. N. Koch and J. K. Allen. Metamodels for Computer-based Engineering Design: Survey and recommendations. *Engineering with Computers*, 17(2):129–150, 2001.

Starossek, U. *Progressive collaps of structures.* Thomas Telford, London, 2009.

Taguchi, G., S. Chowdhury, and Y. Wu. *Taguchis Quality Engineering Handbook.* Wiley, New York, 2004.

Wu, D. and J.M. Mendel. Uncertainty measures for interval type-2 fuzzy sets. *Information Science*, 177(23):5378-5393, 2007.

Zang C., M.I. Friswell, and J.E. Mottershead. A review of robust optimal design and its application in dynamics. *Computers & Structures*, 83(4-5):315-326, 2005.