A priori inverse operator estimation for guaranteed error estimate

A. Takayasu[1)], **S. Oishi**[2)], and **T. Kubo**[3)]

[1)]Graduate School of Fundamental Science and Engineering, Waseda University,
Tokyo 169-0072, Japan, takitoshi@suou.waseda.jp

[2)]Faculty of Science and Engineering, Waseda University,
Tokyo 169-0072, Japan, oishi@waseda.jp

[3)]Institute of Mathematics, University of Tsukuba,
Ibaraki 305-8571, Japan, tkubo@math.tsukuba.ac.jp

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Abstract

We consider the guaranteed error estimate method for some two-point boundary value problems. The 'guaranteed' error estimate is rigorous, i.e. it includes all computational error such as the discretization error and the rounding error when solving the problems. The goal of the guaranteed error estimate method is to prove the existence of the exact solution. And the error estimate between the exact solution u and an approximate solution $\hat{u}$ is given as $\|u - \hat{u}\|_X \leq C$, where X is suitable functional space and the constant C is computable. Namely, we can solve the problem with mathematically rigorous. In the proposal method, we first lead the operator equation from the problem by using solution operator. There are two points to compute the constant C. One is the inverse operator estimation. The other is the residual of the operator equation. In the previous work, the inverse operator estimation needs the norm of the inverse matrix, which is a posteriori constant. So we need much time to compute the inverse operator estimation.

Recently, we found the new estimation about the inverse operator estimation. It is given without the norm of the inverse matrix. By using this new estimation, we can estimate the inverse operator as a priori constant. We expect new estimation to improve the speed of the guaranteed error estimate method. Because we can get the constant C without computing the norm of the inverse matrix. On the other hand, it needs a condition to use the new estimation. In this paper, we introduce new estimation for the inverse operator estimation. And we also show the condition to use a priori estimation.

References

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