Finite Element Structural Analysis using Imprecise Probabilities Based on P-Box Representation

H. Zhang[1)], **R. L. Mullen**[2)], and **R. L. Muhanna**[3)]

[1)]University of Sydney, Sydney 2006, Australia, haozhang@usyd.edu.au

[2)]Case Western Reserve University, Cleveland 44106, USA, rlm@case.edu

[3)]Georgia Institute of Technology, Atlanta 31407, USA, rafi.muhanna@gtsav.gatech.edu

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Abstract

Imprecise probability identifies a number of various mathematical frameworks for making decisions when precise discrete probabilities (or PDF) are not known (Walley, 1991). Imprecise probabilities are normally associated with epistemic uncertainty where the available knowledge is insufficient to construct precise probabilities. While there is no "unified theory of imprecise probabilities" (Walley, 2000), most frameworks describe the uncertainty in terms of bounded possibilities or unknown probabilities between a specified lower and upper bounds. In this paper, we will present a finite element method that employs the concept of P-boxes to represent imprecise probability information (Ferson, et al., 2003). P-boxes represent interval bounds on the cumulative probability distribution function (CDF). Any CDF that is contained within the bounded region is a possible CDF for the random variable. Thus, the p-box approach provides a general mechanism for handling problems with a mixture of imprecise probability and conventional probabilistic information.

In this paper, the authors' previous developed interval finite element methods (Muhanna and Mullen, 2001) are extended to compute p-box structures of a finite element solution given stiffness and loading parameters described by p-boxes. Both direct and interval Monte Carlo algorithms are presented along with exemplars problems that illustrate the capabilities of the new methods. The computational efficiency of the p-box finite element method is also presented.

References

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