On Improvement and Deterioration of a Repairable System Under Generalized Stochastic Orders

Asok K. Nanda[1], and **Amarjit Kundu**[2]

[1]Department of Mathematics,
Indian Institute of Science Education and Research Kolkata,
Mohanpur Campus, Mohanpur 741252, West Bengal, India, asoknanda@yahoo.com

[2]Department of Mathematics, Santipur College,
Santipur, Nadia-741404, West Bengal, India, bapai_k@yahoo.com

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Abstract

Any repairable system is said to improve (deteriorate) with the passage of time if the interarrival times of failure tend to get larger (smaller) in some sense. The comparison of interarrival times in terms of different partial orders viz. LR (Likelihood Ratio), FR (Failure Rate), ST (Stochastic), MRL (Mean Residual Life) orders has been discussed in the literature. Nanda (1997) has compared the interarrival times with respect to s-FR and s-ST orders so that many results of the literature become the particular cases. It is to be mentioned here that, for $s = 0, 1, 2$, s-FR order gives LR, FR and MRL orders respectively, whereas for $s = 1, 2$, s-ST order reduces, respectively, to ST and $HMRL$ (Harmonic Mean Residual Life) orders. In this paper improvement (deterioration) of a repairable system in the sense of several generalized stochastic orders has been discussed.

Let $\{S(n), n = 1, 2, \ldots\}$ be a sequence of nonnegative random variables such that $0 \equiv S(0) < S(1) < \cdots < S(n) < \cdots$. $S(n)$ is defined as the time of n-th failure. Define $X(n) = S(n) - S(n-1)$, $n = 1, 2, \ldots$, and $N(t) = \sup\{n : S(n) \leq t\}$. Then $X(n)$ denotes the time between $(n-1)$-th failure and n-th failure, and $N(t)$ the number of failures by time t. Here we assume that $X(0) \equiv 0$. Let $X_{s_{n-1}}(n)$ denote the random variable $[X(n)|S(n-1) = s_{n-1}, S(n-2) = s_{n-2}, \ldots, S(1) = s_1]$, where $0 < s_1 < s_2 < \cdots < s_{n-1}$ are real numbers, and $n = 2, 3, \ldots$. For $n = 1$, the random variable is taken as $X(1)$.

Here we prove some results when $X_{s_n}(n+1)$ is compared with $X_{s_{n-1}}(n)$ in s-FR, s-ST and s-CX sense. We also compare $X_{s_n}(n+1)$ and $X(1)$ with respect to these orders. We also prove some results on shifted generalized stochastic orders. In this paper simulation of the optimal replacement time for different reliability distributions are also given.

References

[Nanda1997] Nanda, A. K. On Improvement and Deterioration of a Repairable System. *IAPQR Transactions*, 22(2):107-113, 1997.