A Verified Automatic Contour Integration Algorithm

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Abstract

This talk is concerned with the numerical integration on the complex plane. In this talk, a verified automatic contour integration algorithm using numerical computations is proposed. The proposed algorithm is an automatic integration algorithm based on the trapezoidal rule. Although error bounds of the formula depends on the area on the complex plane, it is difficult to specify the shape of that area. A theorem based on the error analysis of the formula for the periodic function is useful for the problem, and then we propose the algorithm based the theorem. By the proposed algorithm, it is possible to calculate the upper bound of the error, so that it can be verified that the result of the contour integration satisfies a given tolerance. Finally, numerical results are presented showing the performance of the proposed algorithm.

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