Exact Determinant of Integer Matrices

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Abstract

Computing a determinant of a given matrix has several applications. For example, it is used for computational geometry (cf. e.g. Emiris and Pan (2001)), in which it is essential to check the sign of the determinant. Moreover, it is also used for robust control methods (cf. e.g. Smagina and Brewer (2002)).

In this paper, we restrict A to be an integer matrix. Then the determinant of A is also an integer. Such a case has been discussed in Pan and Yu (2001); Kaltofen and Villard (2004); Emiris and Pan (2005) and the literatures cited there. Of course, we can efficiently obtain the approximation $\widetilde{d}$ of $\det(A)$ via LU factorization of A by floating-point arithmetic. The aim of the paper is to obtain the *exact* value t of $\det(A)$, where t is some integer. For this purpose, we try to compute a rigorous enclosure $[d] = [\underline{d}, \overline{d}]$ such that $\underline{d} \leq \det(A) \leq \overline{d}$ with $\lceil \underline{d} \rceil = \lfloor \overline{d} \rfloor = t$, which implies $\det(A) = t$, by use of verified numerical computations. It is much more difficult than to simply compute $\widetilde{d}$, since we need to obtain a verified and accurate result of the determinant.

Let $\mathbf{u}$ be the unit roundoff. In IEEE standard 754 double precision, $\mathbf{u} = 2^{-53}$. Let $\kappa(A) := \|A\| \cdot \|A^{-1}\|$ be the condition number of A. In this paper, we propose an algorithm of computing an accurate and verified enclosure of $\det(A)$ with A being extremely ill-conditioned, i.e. $\kappa(A) \gg \mathbf{u}^{-1}$. The proposed algorithm uses an accurate matrix factorization presented in Ogita (2009), which is based on standard numerical algorithms and accurate dot product.

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