On Correlation Control in Monte Carlo type Sampling

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Keywords: *Latin Hypercube Sampling; covariances; multivariate random vectors; dependence; copula; statistical correlation distribution; uncorrelatedness; orthogonality; orthogonal vectors; combinatorial optimization; Simulated Annealing*

Abstract

The objective of this paper is a study of performance of various techniques for correlation control when sampling from a multivariate population within the framework of Monte Carlo simulations, especially Latin Hypercube Sampling. In particular, we study the ability of the methods to fulfill the prescribed marginals and correlation structure of a random vector for various sample sizes. Two norms of correlation error are defined, one very conservative and related to extreme errors, other related to averages of correlation errors. We study behavior of Pearson correlation coefficient for Gaussian vectors and Spearman rank order coefficient (as a distribution-free correlation measure).

The paper starts with theoretical results on performance bounds for both correlation types in cases of desired uncorrelatedness. It is shown that, under some circumstances, a very high rate of convergence can theoretically be achieved. These rates are compared to performance of other previously developed techniques for correlation control, namely the Cholesky orthogonalization as applied by Iman and Conover; and Owen's method using Gram-Schmidt orthogonalization. We show that the proposed technique based on combinatorial optimization yields much better results than the other known techniques.

When correlated vectors are to be simulated, a recently proposed technique exhibits nearly the same excellent performance as in the uncorrelated case provided the desired vector exists. It is shown that the technique provides much wider range of acceptable correlations than the widespread Nataf model (known also as the Li-Hammond model or the NORTA model) and that it is also much more flexible than the Rosenblatt model.

References

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